About the Journal

Teachers and Curriculum is an online peer-reviewed publication supported by Wilf Malcolm Institute of Educational Research (WMIER), Faculty of Education, The University of Waikato, Hamilton 3240, New Zealand. It is directed towards a professional audience and focuses on contemporary issues and research relating to curriculum pedagogy and assessment.

Notes for Contributors

Teachers and Curriculum welcomes:

- research based papers with a maximum of 3,500 words, plus an abstract or professional summary of 150 words, and up to five keywords;

- opinion pieces with a maximum of 1500 words; and

- book or resource reviews with a maximum of 1000 words.

Focus

Teachers and Curriculum provides an avenue for the publication of papers that

- raise important issues to do with the curriculum, pedagogy and assessment;

- reports on research in the areas of curriculum, pedagogy and assessment;

- provides examples of informed curriculum, pedagogy and assessment; and

- review books and other resources that have a curriculum, pedagogy and assessment focus.

Submitting articles for publication

Please consult with colleagues prior to submission so that papers are well presented. Email articles to T&C Administrator, Wilf Malcolm Institute of Educational Research, wmier@waikato.ac.nz.

Length: Articles should not normally exceed 3,500 words (including references); shorter articles such as research reports, poetry, short stories or commentaries are welcome.

Layout and number of copies: Text should be double-lined spaced on one side of A4 paper with 20mm margins on all edges. Font = Times New Roman for all text and headings. All headings must be clearly defined. All tables, figures, diagrams or photos should be submitted separately. The article should clearly show where each is to appear within the text. All submissions must be submitted as word documents. Only the first page of the article should bear the title, the name(s) of the author(s) and the address to which reviews should be sent. In order to enable 'blind' refereeing, please do not include author(s) names on running heads.

Foot/End Notes: These should be avoided where possible; the journal preference is for footnotes rather than endnotes.

Referencing: References must be useful, targeted and appropriate. The Editorial preference is APA style; see *Publication Manual of the American Psychological Association* (Sixth Edition). Please check all citations in the article are included in your references list and in the correct style.

Covering letter: When submitting a manuscript to *Teachers and Curriculum,* authors must, for ethical and copyright reasons, include in a covering letter a statement confirming that a) the material has not been published elsewhere, and b) the manuscript is not currently under consideration with any other publisher. A fax and email contact should also be supplied.

Editorial: All contributions undergo rigorous peer review by at least two expert referees. The Editors reserve the right without consulting the author(s) to make alterations that do not result in substantive changes. The Editors' decisions about acceptance are final.

Copyright: Publication is conditional upon authors giving copyright to the Faculty of Education, The University of Waikato. Requests to copy all or substantial parts of an article must be made to the Editors.

Acknowledgement of Reviewers

The Editors would like to acknowledge the contribution of the reviewers.

TABLE OF CONTENTS

MATHEMATICS IN STUDENT-CENTRED INQUIRY LEARNING: STUDENT ENGAGEMENT

NIGEL CALDER
Faculty of Education
University of Waikato

Abstract

This paper examines how mathematical understandings might be facilitated through student-centred inquiry. Data is drawn from a research project on student-centred inquiry learning that situated mathematics within authentic problem-solving contexts and involved students in a collaboratively constructed curriculum. A contemporary interpretive frame was utilised and mixed methods were used to collect data. The project took place with a Year 10 class in a purpose-built New Zealand secondary school. The findings indicated that mathematics centred on real-life learning was highly engaging, with student choice and the co-construction of their research questions central to that engagement.

Key words

Inquiry learning, mathematics, student-centred, engagement, secondary school

Introduction

Having mathematical and statistical literacy is essential to being an effective participant in an increasingly globalised world. There is a diverse range of everyday practical situations where participants draw on mathematical thinking. Frequently in traditional secondary mathematics classrooms, the contextualisation of mathematical knowledge is considered as an application of learnt skills, rather than an aspect of the initial engagement. However, rather than commencing with particular skills or definitions to be applied later, some mathematics educators contend that the learning should be initiated by rich contexts that require mathematical organisation—contexts that can be mathematised (Freudenthal, 1968; van den Heuvel-Panhuizen, 2010). Some researchers contend that learning is enhanced when the students have some ownership of process and meaning (e.g., Wenger, 1998). In order to establish such ownership, teachers often attempt to design problem-solving contexts that incorporate students' individual naturalistic out-of-school experiences and perspectives into the learning situation (Lowrie & Clancy, 2003).

Affective dimensions of the process of learning in mathematics, including motivation and task persistence, are also more likely to be enhanced in authentic situations (Brough & Calder, 2012). The classroom environment and the learning culture that the teacher develops likewise influence the ways understanding evolves. De Corte, Verschaffel and Greer (2000) maintained that in order for students to make meaningful connections to real-life contexts when solving problems, they needed to be immersed in innovative learning environments that utilize teaching/learning processes that differ substantially from traditional classroom practices. They proposed that tasks should be well structured, diverse and authentic. Authentic tasks reflect the nature of real-life problems because they are complex and contain multiple perspectives. They offer multiple pathways to investigate, are frequently open-ended, and may include a range of conditional solutions.

Student-centred inquiry learning is a democratic teaching approach where meaningful contexts are central to both individual and class learning trajectories. Students pose questions, issues or inquiries that are of genuine interest to them and curriculum is collaboratively co-constructed (Beane, 1997). Subject knowledge is integrated within student-initiated inquiry. For example, statistics may be employed to organise and explore data, such as, students making tables and then graphing the heights of athletes in relation to the Olympic records for the 100m and 200m sprints. Student-centred inquiry might also evoke the need for explicit teaching on, for example, proportions or the types of graphs to best analyse and compare relationships, measurements or costs.

Educators interested in curriculum integration are also suggesting that student-centred inquiry, based on problems that the students pose, can lead to enhanced student ownership, engagement and

understanding (Beane, 1997; Brough, 2012; Dowden, 2010). Here, engagement is seen as being actively occupied with a task or activity. The New Zealand Curriculum (Ministry of Education, 2007) advocates an authentic inquiry approach that facilitates high-level and critical thinking. Learning in mathematics should promote thinking. It should facilitate students' ability to think in logical, creative, critical and strategic ways (Ministry of Education, 2007).

Meanwhile, Lin (2005) contends that the setting and justification of informal conjectures is a rich vein for developing mathematical thinking. The ways that students make initial sense of an investigative situation, and how subsequent learning trajectories are conditioned by those initial exchanges, influences the manner in which their generalisations and informal conjectures develop. For example, generalisations about the commutative property (a+b = b+a), if engaged through adding money rather than more abstract numerical examples, and informal conjectures such as the sum of two odd numbers is an even number, could develop differently through varying contexts. These, in turn, filter the conjectures that emerge, the patterns the students perceive, and the student understanding (Calder, 2011). Learning emerges through the posing and interpretation of the students' inquiry questions from their current perspectives, engagement with the situation, and reflection. This process leads to a modified perspective from which further evolving interpretations and understandings are made. As learners re-engage with tasks, informal mathematical conjectures often have their beginnings (Calder, Brown, Hanley, & Darby, 2006). Other researchers have noted that the development of mathematical conjecture and reasoning can be derived from intuitive beginnings (Bergqvist, 2005; Dreyfus, 1999).

Meanwhile, processing mathematics through digital pedagogical media has been shown to enhance students' ability to model mathematically (Zbiek, 1998). Authentic mathematical inquiries, for example, using WebQuests to investigate tessellations, have been enhanced by the use of digital pedagogical media to research and analyse the inquiry questions (Salsovic, 2009; Calder, 2011). This paper reports on a research project that used a contemporary interpretive lens, to analyse a single-class case study considering the ways that Year 10 students engaged with mathematics learning through the student-centred inquiry process, and how this might influence their learning. In particular, the ways that student-centred, authentic problem posing influences student motivation to engage with mathematics was investigated. The researcher contends that the engagement led to enhanced learning opportunities.

Methodology

A contemporary interpretive approach was utilised to analyse the data, with learning seen as a process of interpretation, and understanding as an ongoing process rather than a fixed reality. Our understandings evolve through cyclical interpretations with the mathematical phenomena and the constant drawing forward of prior experiences and understandings. The pedagogical medium, the mathematical task, the pre-conceptions of the learners, and the associated dialogue evoked are interdependent and it is from their relationship with the learner that understanding emerges. Understanding emerges from cycles of interpretation, but this is forever in transition: there may always be another interpretation made from the modified stance (Calder, 2011).

Mixed methods were used to collect data, including semi-structured interviews with student groups and the class teacher, informal discussions (electronic and face to face), student blogs, naturalistic observations, work samples, and photographs. One Year 10 class was involved, with students aged from 13 to 15 years. Their secondary school is decile 5 and situated in a provincial city.

The students were in the second year of learning through a student-centred inquiry approach that incorporates curriculum content within the investigation of an inquiry question. The students posed this fertile question and the inquiry was co-constructed with the teacher. In this case the inquiry was focused around the topic of the Olympic Games, while also incorporating the statistical inquiry cycle. The students also had content-specific lessons that usually evolved from questions that emerged from the inquiry process. A case-study approach was used to document this month-long inquiry. The ways mathematical learning emerged from the inquiry questions the students posed and explored, and how the exploration of these questions influenced their approach, the learning process undertaken and the students' mathematical understanding were examined.

Results and discussion

As the data was organised and preliminary analysis took place, a number of themes began to emerge. This paper is concerned with the students' engagement and their approach to the learning. The inquiry process is generally described as a cyclical evolving spiral of five stages: wondering, exploring, organising, presenting, and evaluating. When the students posed their inquiry questions within personal interests, their curiosity and personal intrigue were stimulated, leading to enhanced engagement and motivation. This interest may have been from direct involvement in the sport, through a media stimulated engagement such as watching it on TV, or by personal intrigue around a particular aspect e.g., the scoring system in archery. Student comments below were indicative of these aspects:

> I chose my sport and research question based on my personal interest. I enjoy running and liked watching the running events on TV.

> I chose my maths question to investigate because of my personal interest in diving. It is a big event at the Olympics.

> I got the idea from a news item on Valerie Adams. I really liked watching her doing the shot put.

> I started from the guidelines we were given for the inquiry. I chose it from personal interests and things I liked watching. I had some prior knowledge about it, but I wanted to find out more.

Some students were initially negative about the inquiry process in mathematics but became very positive due to investigating within an area of personal interest. The following student was very negative about the whole process at first. They didn't want to engage and indicated that they were negative about mathematics in general.

> I just don't really want to do it. I'm not sure what to do but I'm not really interested.

They became more engaged after they had identified a sport. Thirty minutes later:

> I'm interested in BMX. I like riding BMX—I do it lots of the time.

As recorded in the observational data, once they had begun the research process, because of their personal interest, they became very animated and engaged:

> Look, I've found these really cool websites and started making a table up of the results. My cousin in Hamilton knows Sarah Walker really well so I'm going to text him and her too. I've found the email of her coach and trainer on the website so I'll email them to get information.

Another student initially said they preferred doing mathematics by bookwork and worksheets, but their attitude and engagement changed through the process of researching a question they had posed in a context they were interested in. The following observations were recorded:

> It's interesting how they can transition too. Like Pia was really reluctant at first. She wasn't enjoying it and wanted to do worksheet or bookwork. Then later in the week she said: I'm really loving this now. I've got my 100m stuff. It's what I'm interested in. I've got all my data now—I've done my research—I'm really learning this now. It's going really well. Today she said: I've made sense of my questions—I've answered them and I'm moving into something else.

The differentiation of the learning was supported effectively by the use of needs-based workshops, both teacher and student generated. The teacher, through her knowledge of the students, allied with her content and pedagogical content knowledge, anticipated concepts (e.g., box-and-whisker graphs) or processes (e.g., formulating an inquiry question) that some students would require scaffolding. She gave opportunities for students to gather at her teaching table to consider these processes or particular content. Other needs-based workshops were student initiated. They were in more informal groupings that generally arose from student self-identification and self-referral. The teacher made the following comments in her interview.

Some of them didn't know how to work out volume but I'd call them when they had identified it was something they needed to know, to learn. They're still working on the inquiry project but they identify a skill that they need and I, or someone in the class, teaches them it. Sometimes they can find how to do it on the web, especially YouTube.

The students were very comfortable with this self-identification and generally had no hesitancy with joining groups when they were offered. This was different to the researcher's observations in other Year 10 classes, where students had been hesitant to acknowledge their lack of understanding so publicly and had often been reluctant to come forward to such workshops.

The manner in which the mathematical elements emerged from the inquiry process was central to the purpose of the research project. The inquiry facilitated the identification of mathematical concepts and processes that individual students required. Some of these were:

It helped me remember mean, median, mode and gave a purpose to use them—I used the range too.

We used statistics and graphs in our inquiry—mean, median, mode, range. We used percentage too.

Other data also linked the conceptual knowledge with the associated process, some of which was new material that arose out of the inquiry process:

We needed to find out more about box-and-whisker—how do it, how it worked.

We used statistics and graphs, and we had to find out about the methods that you needed.

You need to figure out how to put it into graphs and use the software.

Others indicated the use of digital technology for researching content and processes:

To find out how to do stuff we researched on the net—how to do tables, graphs with the computer. We also tried to find how to do box-and-whiskers on the computer.

We needed to work out how to use spreadsheets and Excel.

I had to research the methods that I needed, they were related to maths or were statistical.

The Internet was utilised by all students and was identified as the first and most productive source for research. The teacher had identified and shared a valuable broad-based website that included productive links. For some of the students this facilitated a more direct, focused engagement with the Internet. The students indicated that they preferred not to just surf or browse through the research and analysis phases, but were specific about their use of software and websites, and discriminatory regarding the matching of software and websites to their particular requirements. Some, with permission, also used their phones and texting effectively for contacting external sources.

Interestingly, while most students and the teacher noted that the inquiry often took them to new content or processes, one student stated that

the maths in inquiries is generally basic or we don't really use it [maths] at all.

However, the student blogs indicated the high perception of utilising statistics in the inquiry. Many used tables and graphs effectively for sorting and analysing the data. They found this effective for the critical thinking approach, compare and contrast. For example, from the blogs:

I am analysing, taking all the statistics and comparing them and finding the mean and median.

Read, interpret, understand, and make tables; and use spreadsheets.

Putting our information into a table and then creating a graph from our data.

I made a graph. This helped me to compare and contrast the difference between men and women in the throw distances.

Began analysing the info pretty soon—started making a table. It was easy to turn into tables and graphs. This was good for comparing and contrast.

The teacher commented on another form of critical thinking, reasoning and concluding:

> We had one, reasoning and concluding, and they kept saying they hadn't done it but when they looked at what they did with the information and how they'd used it, they had. They can actually be doing it but not know the term for the process, or recognise it.

This indicates the need for the teacher to make both the content and the mathematical processes (including critical thinking) explicit and to accentuate the connections between the mathematical content and other curriculum areas. The newly introduced graph, the box-and-whisker, was used appropriately and effectively in nearly all of the inquiries. Twenty-three of the blog entries made direct reference to it and its usefulness.

The mathematical thinking and knowledge gained through the investigation involved students learning how to create and interpret tables and graphs, how to apply additive and multiplicative strategies to measurements for length, time, and weight, and how to calculate statistics. The learning in the number areas was more application of existing skills rather than the facilitation of new mathematical learning. However, this was an important learning process for many of the students and was also a chance for others to revisit certain mathematical processes, such as computing fractional numbers. Further, some students needed to use proportional thinking and calculate various proportions and statistics and ratios that involved the application of additive and multiplicative strategies. The students reported that they enjoyed the inquiry and the learning that took place. As well as the mathematics discussed, there were cross curricula links to literacy, technology and science. Some moral and ethical questions related to performance-enhancing substances were raised.

Critical thinking, especially the use of compare and contrast, was an important facet of the mathematical thinking that was evoked. It was prevalent in analysis processes as the students sought trends and patterns in the data. The use of tables and graphs was effective in this process and these were almost exclusively developed in digital form.

A key aspect considered was whether having the mathematics embedded in authentic, student-generated inquiries led to changes in student motivation, engagement, and attitude to doing the mathematical elements of the inquiry tasks—whether or not it enhanced or negatively impacted on their learning experience. The following data were typical of the responses to the question, Did doing the maths within an investigation change the way you felt about doing maths? If so, in what ways?

> Let students enjoy [what they are doing], they will do it. The chance to pick your own groups, to choose. It meant we were more motivated and on task. But you need to be working with the right person. It helps you to keep going longer.

> The work was relevant to the subject. It felt good.

> Different, but had some advantages—I liked the choice. I felt motivated and enjoyed doing the maths and I found it more interesting. I get more done working this way and what we did in maths helped us with our main inquiry.

> We wanted to do it—it was fun. There is less effort with writing. I felt it was useful being able to do what you want and work with friends. I enjoyed the starting activities and finishing off with the conversation.

> I thought to pose your own question was really important. It motivated you because you're involved in it.

> Applies to real life, we chose the sport that we liked, and are interested in, to find out about.

The sense of motivation was hinged to personal choice—of the research question, the inquiry process and the people to work with. The purpose of what they were doing was clearer to them, and the relevance to everyday life was also mentioned several times. Situating the statistical inquiry in a high interest theme that was high profile in the media and being encountered consistently at home, socially and at school helped the students to see the purpose of the mathematics they were doing. It also embedded their understanding of the mathematics and the applicability of the statistical processes to other situations. Several students indicated that they planned to use tables and graphs of statistics in other inquiries. Many of the students were able to self-manage the process, albeit with engagement

with other students, the teacher and other experts regarding specific content or processes. A more scaffolded approach was required for some students, while specific content or process workshops were critical for overall class engagement.

Conclusions

In student-centred inquiry, teaching takes place "just in time" as students require skills to solve particular problems. This enables students to identify the purpose for the acquisition of skills. Not only does the purpose become apparent, the contextualisation of the content appears to embed the students' learning and the understanding better. While teachers of student-centred inquiry learning are required to assume a more facilitative and empowering approach to teaching, extending student's mathematical thinking into areas they may not have considered and explicit teaching are still vitally important. In this instance, introducing the students to box-and-whisker graphs extended their mathematical knowledge and was identified as a tool that might be effectively used in future inquiries.

Teachers need to consider the ways in which critical thinking emerges and how it is enhanced—compare and contrast, reasoning and drawing conclusions, creative thinking/creating metaphors, and metacognition are types of critical thinking that need to be made explicit to students, as do their links to mathematical thinking. Mathematical thinking includes the generalisation and distillation of central concepts. Using the critical thinking processes of compare and contrast, as well as reasoning and drawing conclusions, relate directly to making generalisations.

Another aspect of note within the inquiry process was the comfort and ease with which students moved to and engaged with the needs-based workshops. These were open forums that addressed content and process elements that some students wished to understand further. They encompassed a variety of social and ability groups. Compared to observations of other classes in more traditional settings at this age level, the students were very relaxed about joining them and approached them in a positive manner. They were an effective mechanism for addressing, in a timely manner, particular mathematical content and process questions for specific learners.

A high level of motivation and engagement was witnessed throughout the inquiry. Subject material was repositioned contextually and learning strengthened as students were motivated to acquire the skills and knowledge necessary to solve relevant problems (Brough & Calder, 2012). The majority of students indicated that they enjoyed learning mathematics through the inquiry process. This generally has positive ramifications for the learning process. There were strong indications that the high level of student engagement was a result of student involvement in determining the learning context and research question. This enhanced their engagement and motivation, as well as providing an opportunity to explore an issue with which they had personal interest or curiosity. Of particular note, were two students who were initially very negative about engaging with the process, but once they had selected their sport (both had a strong personal involvement with these sports) became interested, and by the end of the first block were fully engaged and enthusiastic about the process. These two particular situations exemplified the notion of cyclical interpretation, which might lead to engagement and understanding. As these two students' initial perspectives were transformed through personal interpretations and questions, their interpretation of the task also changed. This led to re-engagement with the task from modified perspectives. They began to employ mathematical processes positively and with purpose, until their inquiry question was resolved through several iterations of cyclical interpretation.

Implications

Given the findings of this small study, schools need to consider structuring their timetables to allow longer blocks of time for students to research and engage more fully with their inquiry. Teachers need to consider themselves as inquirers as well as facilitators—they need to be willing to take some risks and release some of the control over content knowledge and processes. They require appropriate content knowledge to recognise the potential mathematics learning with the students' inquiries and to extend children into new subject material. Relational knowledge of their students is central to knowing the types of learning approaches and interests that will "hook" their students into inquiry and shape the ongoing learning experiences. The front-loading of an inquiry framework is critical for initial engagement. Teachers need to use images, video clips or an interesting, humorous or gruesome

anecdote. It is important that the teams of teachers in each common learning area utilise specialists within their planning groups so as to recognise and optimise potential learning opportunities.

There were several limitations to the study in regard to examining the research question. The use of statistics as the mathematical topic, although embedded within the inquiry, meant that the statistical inquiry cycle was automatically part of the mathematical learning process that the student inquiries evoked. This naturally positioned the mathematics within an inquiry process. However, previous student inquiries at the school have provided rich sources of mathematical thinking in geometry, measurement and number—in particular, those that considered nets of bodies, and one that led to the actual development of a mountain bike track.

The research revealed related aspects that could be examined in future research. One aspect to consider is whether the student-centred inquiry process needs to be different, or transform in some way, with different age groups. A second aspect for further examination is the use and effectiveness of self-assessment and self-review processes. Although these are inherent in the nature of the inquiry process, the extent to which they might be recognised and formalised is worthy of further examination. Thirdly, an exploration of the ways in which inquiries might better facilitate algebraic thinking and mathematical modelling would add considerably to the understanding in this area.

References

Beane, J. (1997). Curriculum integration: *Designing the core of democratic education*. New York, NY: Teachers College Press, Columbia University.

Bergqvist, T. (2005). How students verify conjectures: Teachers' expectations. *Journal of Mathematics Teacher Education, 8,* 171–191.

Brough, C. (2012). Implementing the democratic principles and practices of student-centred curriculum integration in primary schools. *Curriculum Journal, 23*(3), 345–369. Retrieved from http://dx.doi.org/10.1080/09585176.2012.703498

Brough, C., & Calder, N. S. (2012). Mathematics as it happens: Student-centred inquiry learning. In J. Dindyal, L. Cheng, & S. Ng (Eds.), *Mathematics education: Expanding horizons. Proceedings of the 35th Annual Conference of the Mathematics Education Group of Australasia* [eBook] (pp. 138–145). Singapore: MERGA.

Calder, N. (2011). *Processing mathematics through digital technologies: The primary years*. Rotterdam, The Netherlands: Sense.

Calder, N., Brown, T., Hanley, U., & Darby, S. (2006). Forming conjectures within a spreadsheet environment. *Mathematics Education Research Journal, 18*(3) 100–116.

De Corte, E., Verschaffel, L., & Greer, B. (2000). Connecting mathematics problem solving to the real world. In *Proceedings of the International Conference on Mathematics Education into the 21st Century: Mathematics for living* (pp. 66–73). Amman, Jordan: The National Center for Human Resource Development.

Dowden, T. (2010). Curriculum integration in New Zealand: Rediscovering the potential of student-centred curriculum design in the middle years. *Australian Journal of Middle Schooling, 10*(2), 4–10.

Dreyfus, T. (1999). Why Johnny can't prove. *Educational Studies in Mathematics, 38,* 85–109.

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics, 1,* 3–8.

Lin, F. (2005). Modelling students' learning in argumentation and mathematics proof. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 1, pp. 3–18). Melbourne, VIC, Australia: PME.

Lowrie, T., & Clancy, S. (2003). Multimodal texts: Numeracy development in naturalistic learning contexts. In *Proceedings of the Annual Australian Association for Research in Education Conference, Auckland, New Zealand* [CD-Rom]. Retrieved from http://www.aare.edu.au/publications-database.php/3404/multimodal-texts-numeracy-development-in-naturalistic-learning-contexts

Ministry of Education. (2007). *New Zealand Curriculum*. Wellington, New Zealand: Learning Media.

Salsovic, A. (2009). Designing a WebQuest. *Mathematics Teacher, 102*(9), 666–671.

Van den Heuval-Panhuizen, M. (2010). Reform under attack—Forty years of working on better mathematics education thrown on the scrapheap? No way! In L. Sparrow, B. Kissane, & C.

Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia*. Freemantle, WA, Australia: MERGA.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, England: Cambridge University Press.

Zbiek, M. (1998). Prospective teachers' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education, 29*(2), 184–201.