Modelling Operations: Counting-based and Collections-based approaches to Computation

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ABSTRACT: This paper focuses on different ways of modelling operations in mathematics; namely, counting-based and collections-based models.

The two models are examined in relation to the literature on diagramme literacy, as well as New Zealand's mathematics curriculum document.

Implications for the Numeracy Development Project and the New Zealand Number Framework are also considered.

The value of presenting collections-based models in addition to the (counting-based) empty number line models in materials for teachers is discussed.

Splitting diagrammes are presented as a collections-based alternative to empty number line models.

According to Boerst and Schielack (2003), "computation is a core element of mathematics, school curricula...and...the knowledge that is important for day-to-day living in the real world" (p. 292). Two different conceptions of number have been identified as underpinning the solutions developed by children to solve problems involving operations: counting-based and collections-based solutions (Yackel, 2001). Education systems differ markedly in their approach to arithmetic practices, some emphasising counting and others focusing more on collections (Anghileri, 2001a). For example, the Dutch see counting as fundamental to the calculating procedures; place value is not referred to explicitly, and there is a more holistic approach to number with written calculation strategies retaining the values of the whole numbers throughout (Anghileri, 2001a). The English, on the other hand, take a more collections-based approach, with the collections organised according to the value of the units (e.g., hundreds, tens & ones).

Counting-based (or sequence-based) solutions are based on the number line, and begin with one of the numbers in the problem followed by jumps either forwards (in the case of addition) or backwards (in the case of subtraction), depending on the particular operation involved. These approaches are often referred to as "complete number methods" (or N10) because the first number is kept intact (Beihuizen, 2001). Even when there is no evidence of counting per se, it is assumed that abbreviated counting (e.g., counting on) is the basis for the solution (Yackel, 2001). The empty number line which developed out of Realistic Mathematics Education in the Netherlands is a good example of a counting-based solution (Beishuizen, 1999; Carr, 1998).

Collections-based solutions are based on the partitioning of numbers into component parts and the subsequent joining (in the case of addition) or separating (in the case of subtraction) of the parts to get the answer (Yackel, 2001). These approaches have been referred to as "split number methods" (split tens or 1010), because of the ways that numbers are partitioned into tens and ones (e.g., 25 as two tens and five ones), and the units of each type are combined separately before being put together to give the answer (Yackel, 2001). Resnick (1983) has called this kind of partitioning "unique partitioning" to distinguish it from the various "multiple partitionings" which are also possible (e.g., 25 as one ten and 15 ones). However, a collections-based solution using partitioning and recombining does not necessarily require standard place-value partitioning, although many writers seem to make this assumption (e.g., Yackel, 2001). In England, for example, place value is seen as an important organizing principle.

Some writers take the view that counting-based methods are more primitive than collections-based methods, and that children will eventually need to move away from counting-based to collections-based solutions (e.g., Resnick, 1983; Young-Loveridge, 2001, 2002a, 2002b). However, the outstanding success of Dutch students on international comparisons of mathematics achievement (In the TIMSS study, the Netherlands was first or second out of all the non-Asian countries participating) throws some doubt on the idea that a counting-based model is inferior to a collections-based model for facilitating mathematics learning. Yackel (2001) argues that it is important for children to have both a collections-based and a counting-based conception of number because of the flexibility that it gives them in terms of possible solution strategies.

Yackel's View of Counting based or Collections based Models

Erna Yackel (2001) has written about the differences between counting-based and collections-based models. She argues that "it [is] important that children have both a collections-based conception and a counting-based conception of number" (p. 25). Yackel describes the solution strategies of three children who used part-whole strategies (Latanya looked initially as though she was using a part-whole strategy but changed to counting back) (see Figure 1).

Yackel claims that only Dominque's solution is grounded in a collections-based conception, whereas all of the others are grounded in counting-based conceptions. However, I do not agree. I think Dominique's solution reflects an attempt to use the standard written algorithm to solve the problem mentally, by using a so-called "buggy" solution strategy, with the "smaller from larger" bug evident in his

Children's solutions to the problem "52 – 17"

Lawrence: I took away 10 from 52 gives me 42. Then I took away 2 more gives me 40. I have 5 more to take away gives 35.

Latanya: I took 10 from the 52 to give me 42. Then I said, 41, 40, 39, 38, 37, 36, 35.

Denzel: First I took away the 2. Then I took away the 10. Then I took away the other 5. My answer is 35.

Lakisha: First I took away 20 and got 32. Then I put back 3 more and I got 35.

Dominique: First I take 10 from 50 to get 40. Then I take 2 from 7 to get 5. My answer is 45 [sic].

Figure 1. Children's solutions to the problem "53 – 17" reported by Yackel (2001)

subtraction of 2 from 7 (see Brown & Van Lehn, 1982). This kind of mistake is typical of children who try to use the written algorithm without understanding its conceptual basis (e.g. Young-Loveridge, 2002a). Interpreting Dominique's response is made more difficult by the fact that the correct answer to the problem ends in a "5" (see Young-Loveridge, 2002a). It is difficult to distinguish those students who use the faulty "smaller from larger" strategy (e.g., 7 - 2) from those who get the answer using an appropriate strategy (e.g., 12 – 7), but make a small calculation error with the tens. Unlike Yackel (2001), I believe that collections-based conceptions do not need to involve standard place-value partitioning. Observation and discussion with students about their strategy solutions has shown them to use a variety of collections-based (i.e., part-whole) solution strategy involving a series of partitionings and subtractions. The children described in Figure 1 subtracted the parts of the whole using a sequential strategy in a variety of different orders (see Lawrence's & Denzel's solutions in Figure 1). Lakisha, on the other hand, used a compensation strategy.

From Materials to Abstract Entities

Using so-called "concrete" materials to introduce mathematics to children in the early childhood and junior primary years is widely accepted as appropriate practice for students at these levels. However, many teachers of mathematics at the senior primary and secondary levels of the system assume that students at this stage should be able to deal with numbers as abstract entities and make little or no use of concrete materials. Some writers talk about the way that students' thinking moves from a dependence on materials, through a stage where imaging is used, eventually reaching a stage where students can use their understanding of number properties to solve problems in mathematics (see Fuson, 2003; Pirie & Kieran, 1989, 1994). The Numeracy Professional Development Projects have adapted Pirie and Kieran's model and used it as part of a model for teaching students strategies (Ministry of Education, 2003b). The model is presented as a series of concentric circles with "Using Materials" in the centre and "Using Number Properties" in the outer ring. "Using Imaging" appears in a ring between these other two rings. Double arrows are used to indicate that students can move backwards towards more concrete representations, as well as forwards towards more abstract representations.

Recently, some mathematics education researchers have become interested in visualisation and mental imagery. Yackel looks at number concepts and their associations with visual images. According to Yackel (2001), "carefully selected scenarios have the potential of forming an image basis for students' mathematical activity" (p. 27). She talks about the use of single and double ten-frames in helping children to associate particular quantities with visual images. Apparatus or manipulatives are useful calculating aids for developing imagery (Anghileri, 2001a). For example, bead frames or bead strings relate closely to images of counting strategies, as does the empty number line.

Diagramme Literacy

Some writers have warned about the dangers of teaching paper and pencil algorithms before fundamental part-whole thinking is established, claiming that it damages students' developing number sense (see Bass, 2003; Gehrke & Biddulph, 2001; Wright, 2000). Although the primary concern is about the use of written algorithms rather than a problem with written recording per se, Wright suggests somewhat facetiously that "pencil and paper methods should carry a health warning" (p. 7). According to one mathematician (Bass, 2003), traditional algorithms are "cleverly efficient" in that minimal space and writing are used, but they are also "opaque" in that the mathematical meaning of the steps is not clearly evident. Bass asserts that "if these algorithms are learned mechanically and by rote, the opaque knowledge, unsupported by sense making and understanding, often is fragile and error-prone" (p. 326).

Recent writing about so-called "diagramme literacy" identifies it as a component of visual literacy, which is seen as an essential component of students' mathematical development (Diezmann & English, 2001; National Council of Teachers of Mathematics, 2000). Diagramme literacy is about being able to think and learn in terms of images, and to read and write using a shared system of representation. A diagramme can be used as a visual representation to display information in a spatial layout. The advantage of a diagramme is that it can be used to analyse and reveal the structure of a problem, and hence provide a basis for its solution. According to Diezmann and English (2001), the ability to use diagrammes effectively is critical for mathematical thinking and learning. For example, part-whole diagrammes represent the relationships between a part and a whole. Diezmann and English (2001) suggest ways that teachers can facilitate the development of students' diagramme literacy. Yackel (2001) warns that the meaning of diagrammes may be lost on children who do not "see" the critical features of a diagramme in the same way as an adult. Their interpretations are limited by their lack of experience. For example, children may not understand that the sum of the parts of a particular number must equal the original number. Checking children's interpretation of diagrammes is important, lest the rules and procedures for the standard written algorithm be replaced by other rules and procedures for using the tools or diagrammes.

The current New Zealand mathematics curriculum (*Mathematics in the New Zealand Curriculum)* recognises the importance of diagrammes, particularly in relation to mathematical processes skills such as communicating mathematical ideas (Ministry of Education, 1992). Diagrammes are mentioned as one of the ways that mathematical ideas can be recorded and presented to others. The curriculum document also recommends that teachers create opportunities for students to develop good problem-solving techniques by encouraging the use of strategies such as drawing a diagramme, among other techniques (Ministry of Education, 1992).

Number Line Models for Counting–based Processes

The empty number line was designed to provide imagery to encourage and support the development of mental strategies and to help children move away from materials such as cubes, blocks or numbered lines (Beishuizen, 1999). Although the empty number line starts without any numbers, students are encouraged to visualise multiples of ten as "landmarks" which they jump towards or away from (Anghileri, 2001b). The use of bead-strings organised in tens of alternating colours provides a way of introducing the empty number line to students and helps to bridge the gap between materials and the written recoding system used with the empty number line (see Klein, Beishuizen, & Treffers, 1998). The empty number line has gained considerable popularity over the past ten years since Treffers first proposed its use in the Netherlands (Anghileri, 2001a). The empty number line has been included in recent curriculum materials developed as part of reforms in mathematics education in England (Department for Education and Employment, 1999), Australia (New South Wales Department of Education & Training, 2001), and New Zealand (Ministry of Education, 2001a, 2001b, 2002a, 2002b, 2003a).

New Zealand's Numeracy Development Project

An important component of New Zealand's Numeracy Development Project is the Number Framework (see Ministry of Education, 2001a, 2003a). The diagrammes used to show the various part-whole stages in the 2001 materials for teachers were essentially collections-based (see Ministry of Education, 2001b). This was particularly evident at the Advanced Additive/Early Multiplicative stage where an array model was used to show how a 6 x 6 array with an extra row of 6 added on the end could be used to show how 6 x 7 could be derived from knowledge that 6 x 6 = 36. At the Advanced Multiplicative/Early Proportional Part-Whole stage, collections-based diagrammes were used exclusively. To illustrate the associative property, the diagramme showed how a 3 x 27 array could be broken up into 3 arrays of 3 x 9, joined end to end. By rearranging the 3 arrays of 3 x 9, and positioning them one below the other in a block to make a 9 x 9 array, the relationships between various alternative partitionings was clearly evident. The distributive property was also modelling effectively with a 24 x 6 array which was broken down into a 20 x 6 array and a 4 x 6 array side by side.

In 2002, all of the diagrammes in the materials for teachers which had previously been arrays were changed to number line models, reflecting a greater emphasis on counting–based models for solution strategies (see Ministry of Education, 2003a). When comparing collections-based models with corresponding number line models for particular problems, I noticed that certain kinds of problems were better modelled with a collections-based diagramme, while others were better modelled with a counting-based (number line) diagramme.

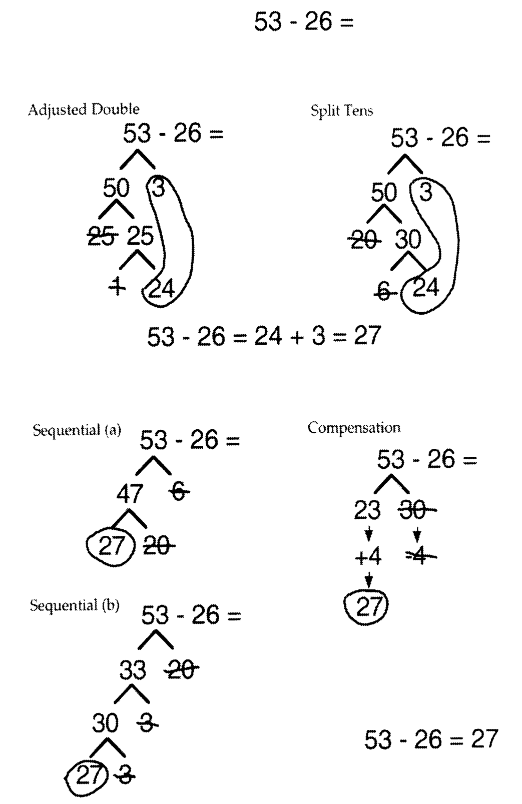

Figure 2. Collections-based Diagrams (Splitting Diagram) for Recording Different Kinds of Solution Strategies for "53 - 26".

Doubles

Problems which can be solved using knowledge of doubles seem to me to be modelled more effectively with a collections-based diagramme than with a number line model. The action of halving involves splitting a quantity down the middle, then making some adjustments to that. For example, in the teachers' materials, at Stage 5: Early Additive Part-Whole, the problem 7 + 8 is given as an example (see Ministry of Education, 2003a). Although the problem can be represented on a number line as two equal-sized jumps of 7 followed by a jump of one, the length of the jumps on a number line is quite arbitrary because the number line is one-dimensional, and the focus is on how far forwards or backwards along the line one needs to go.

A more challenging example is the "bus problem" which has 53 people on the bus and 26 people getting off. Year six children described a range of alternative mental strategies to me. Many of those strategies could easily be represented using a number line model. However, the "Adjusted Doubles" strategy used by some of the children did not easily fit the number line model (see Young-Loveridge, 2002a). These children typically put the 3 of the 53 to one side, split the 50 into two "lots" of 25, took away one 25 and then took another one from the other 25 to make the whole subtrahend 26, then added the remaining 24 to the 3 which had been put aside, making 27 altogether. My attempts to represent this process on a number line were very messy. My collections-based diagramme (a so-called "splitting diagramme") was much more straightforward. Doubles, after all, is the simplest form of multiplication, and hence can be represented by a two-dimensional array.

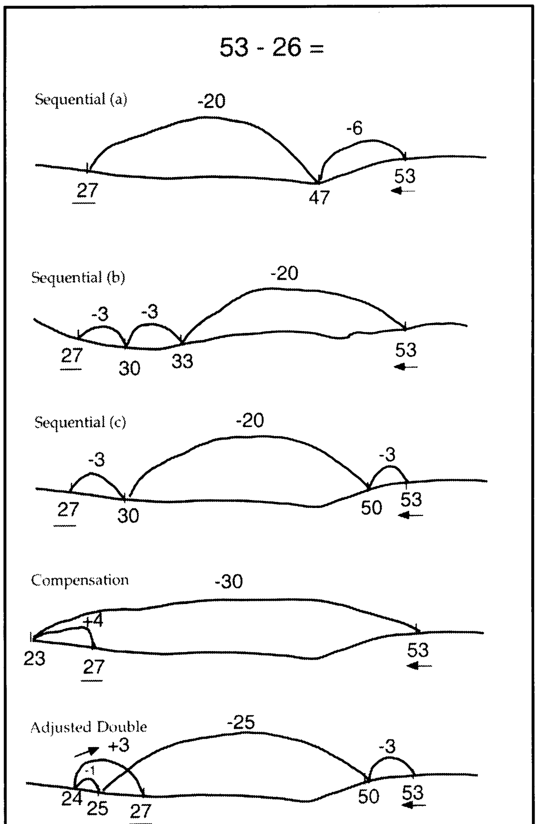

Figure 3. Counting-based Diagrams (Empty Number Line) for Recording Different Kinds of Solution strategies for "53 - 26".

Splitting Diagrammes: A Way to Show Collections-based Solutions

Splitting diagrammes are a way to show the partitioning of numbers in ways that allow the parts to be joined (in the case of addition) or separated (in the case of subtraction) without the need to count by ones (see Gravemeijer et al, 2000; Young-Loveridge, 2001, 2002a). Splitting diagrammes can be used to help children become thoroughly familiar with different ways to partition numbers, and in particular with the various combinations which make ten. Initially, a simple one-step process can be used to help children appreciate the multiple partitioning processes and ways that they can be recorded (see Figures 2 to 5).

Splitting diagrammes provide a valuable way of recording the process of splitting and joining (or separating) parts of numbers, thus reducing the load on working memory. After interviewing hundreds of children and asking them to explain their solution strategies, it became evident that some children experience difficulty in recalling all of the steps they took in solving the problem. Splitting diagrammes offer the chance to record multi-step solution strategies and provide a permanent record of the thinking involved at each of the steps.

This may be particularly valuable for low achieving children who often experience difficulty with working memory (see Gathercole & Pickering, 2000).

Splitting diagrammes were developed as part of an action research project with children in a Year five/six class in a decile one school (Young-Loveridge, 2001). The children had been taught to do multi-digit subtraction across a decade break (e.g., 52 – 17) using the standard written algorithm, but evidence from an earlier study indicated that many children used the faulty "smaller from larger" strategy. I devised a way for students to show the partitioning and separating processes which corresponded to the actual strategies described to me by children in the earlier study. This allowed the children to share a range of different but equally efficient strategies. The idea that there were several equally acceptable ways to solve the problem was in marked contrast to the children's previous experience of one "right" way of working out the answer.

Several children in the group continued to use counting by ones on their fingers to work out the answer, then making the splitting diagramme fit with their calculated answer. A number line model might have been more appropriate for these children to help them appreciate the benefits of working with chunks rather than units of one. The difficulties they experienced with splitting diagrammes are consistent with the finding that students initially use counting strategies and only later progress to the use of part-whole (collections-based) strategies (see, Wright, 2000, Ministry of Education, 2001a,b,c, 2002a, 2002b, 2003).

Closer examination of children's splitting diagrammes revealed that some children did not understand that the sum of the parts should be the same as the original whole (number). A checking process was introduced to make sure that the sum of the parts equalled the whole. Because these students had been taught one "right" way of doing multi-digit addition or subtraction, the idea that there was more than one way of working out the answer to the problem was somewhat puzzling. Some seemed to prefer to work out the answer first using a tried and true method, then generate a splitting diagramme to match this answer. For these children, the splitting diagramme did not seem to be helpful.

Multiplication and Division

Recently I explored ways of using splitting diagrammes to solve multiplication and division problems. It was reasonably straightforward to develop splitting diagrammes for division, particularly when the splits were into halves, thirds, or quarters. However, I quickly found that creating splitting diagrammes for multiplication was not so straightforward. Neither was it easy to show halving and doubling (as in transforming 16 x 4 into 8 x 8), or the corresponding process which involved splitting into thirds then tripling (as in transforming 3 x 27 into 9 x 9). I decided that splitting diagrammes do not work well for multiplication because multiplication is about building up quantities rather than breaking them down.

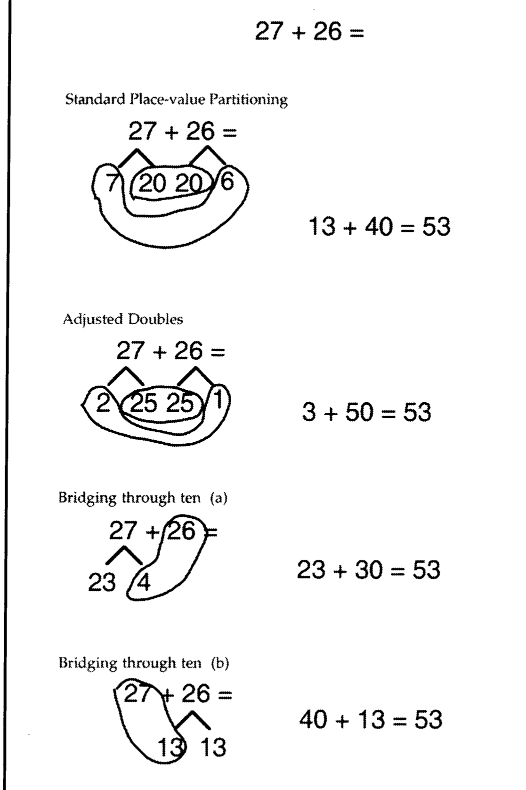

Figure 4. Collections-based Diagrams (Splitting Diagram) for recording Various Different Kinds of Solution Strategies for "27 + 26".

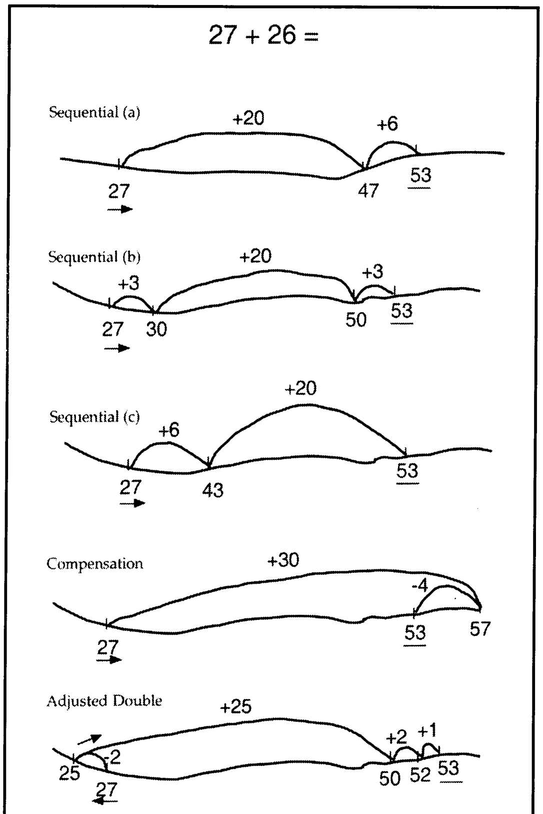

Figure 5. Counting-based Diagrams (Empty Number Line) for Recording Different Kinds of Solution Strategies for "27 + 26".

For this reason I think that array models are in fact better than splitting diagrammes for recording multiplication processes.

When I began comparing the collections-based (array) diagrammes in the 2001 teachers' materials with the counting-based (number line) models in the 2002 and 2003 teachers' materials, I was struck by the fact that when multiplication is represented on a number line, the linearity of the number line inevitably means that the process is depicted as repeated addition (see 16 x 4 in Ministry of Education, 2003, p. 6). Yet we know from literature on multiplication strategies that repeated addition is a lower-level strategy than an array-based approach to solving the problem (e.g., Fuson, 2003; Mulligan & Mitchelmore, 1997).

Conclusion

If our goal is to promote mathematical thinking and help children become flexible problem solvers, then it should be advantageous to show students multiple representations of the same problems. If Erna Yackel is right about the value of both counting-based and collections-based conceptions, then we should be showing students *both* counting-based and collections-based models. It would be good to see the number framework document for the Numeracy Development Project present both empty number line models and collections-based models to represent operations. The array models from the 2001 materials for teachers need to be put alongside the empty number line models to show a more sophisticated conception of multiplication than is possible with the current repeated addition model (see Ministry of Education, 2003a). Students need to be given examples of collections-based as well as counting-based models. However, some problems seem particularly suited to counting-based solution strategies whereas others seem more amenable to a collections-based solution. Problems involving multiplication, even at its simplest (e.g., doubles), seem more suited to collections-based approaches than to counting-based models.

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References

Anghileri, J. (2001a). Intuitive approaches, mental strategies and standard algorithms. In J. Anghileri (Ed.), *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom* (pp. 79-94). Buckingham: Open University Press.

Anghileri, J. (2001b). Contrasting approaches that challenge tradition. In J. Anghileri, (Ed.), *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom.* (pp. 4-14). Buckingham: Open University Press.

Askew, M. (1999). It ain't (just) what you do: Effective teachers of numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 91-102). Buckingham: Open University Press.

Bass, H. (2003). Computational fluency, algorithms, and mathematical proficiency: One mathematician's perspective. *Teaching Children Mathematics*, *9*(6), 322-327.

Beishuizen, M. (1999). The empty number line as a new model. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 157-168). Buckingham: Open University.

Beishuizen, M. (2001). Different approaches, mental strategies and standard algorithms. In J. Anghileri (Ed.), *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom* (pp. 119-130). Buckingham: Open University Press.

Boerst, T. A., & Schielack, J. F. (2003). Toward understanding of "computational fluency". *Teaching Children Mathematics, 9*(6), 292-293.

Brown, J. S., & Van Lehn, K. (1982). Towards a generative theory of "bugs". In T. P. Carpenter, J. M. Moser, & T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 117-135). New Jersey: Lawrence Erlbaum.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*(1), 3-20.

Carr, K. (1998). The empty number line: A good idea for mathematics? *Teachers and Curriculum, 2*, 66-67.

Department for Education & Employment (1999). *National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6.* London: Author.

Diezmann, C. M., & English, L. D. (2001). Promoting the use of diagrammes as tools for thinking. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 77-89). Reston, VA: National Council of Teachers of Mathematics.

Fennell, F., & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7*(5), 288-292.

Flockton, L & Crooks, L. (1998). *Mathematics assessment results 1997: National education monitoring report 9.* Wellington: Ministry of Education.

Fuson, K. C. (2003). Toward computation fluency in multidigit multiplication and division. *Teaching Children Mathematics, 9*(6), 300-305.

Fuson, K. C., Smith, S. T., & Cicero, A. M. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*(6), 738-766.

Garden, R. A. (1996). *Mathematics performance of New Zealand form 2 and form 3 students: National results from New Zealand's participation in the third international mathematics and science study.* Wellington: Research and International Section, Ministry of Education.

Garden, R. A. (1997). *Mathematics & science performance in middle primary school: Results from New Zealand's participation in the third international mathematics and science study.* Wellington: Research and International Section, Ministry of Education.

Gathercole, S. E., & Pickering, S. J. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Education Psychology, 70,* 177-194.

Gehrke, J. & Biddulph, F. (2001). Primary mathematics: It's time we stopped teaching standard algorithms. *Teachers and Curriculum, 5,* 51-54.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modelling, and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Communicating and symbolizing in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 225-274). Mahwah, NJ: Lawrence Erlbaum Associates.

Klein, A. S., Beishuizen, M., & Treffers, A. (1998). The empty number line in Dutch second grades: *Realistic* versus *Gradual* program design. *Journal for Research in Mathematics Education*, *29*(4), 443-464.

Linchevski, L. & Kutscher, B. (1999). Tell me with whom you're learning and I'll tell you how much you've learned: Mixed ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education, 29*, 533-554.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum.

Ministry of Education, (1992). *Mathematics in the New Zealand Curriculum*. Wellington: Learning Media.

Ministry of Education, (2001a). *Curriculum Update: The Numeracy Story.* Wellington: Ministry of Education.

Ministry of Education, Curriculum Division, (2001b). *Early Numeracy Project: Draft teachers' materials*. Wellington: Ministry of Education.

Ministry of Education, Curriculum Division, (2001c). *Advanced Numeracy Project: Draft teachers' materials*. Wellington: Ministry of Education.

Ministry of Education, (2003a). *Book 1: The Number Framework: Numeracy professional development projects 2003*. Wellington: Author.

Ministry of Education, (2003b). *Book 3: Getting Started: Numeracy professional development projects 2003 (Draft)*. Wellington: Author.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28*(3), 309-330.

National Council of Teachers of Mathematics, (2000). *Principles and standards for school mathematics*, Electronic Edition. Reston, VA: National Council of Teachers of Mathematics.

New South Wales Department of Education and Training, Curriculum Support Directorate, (2001). *Moving the numeracy boundary*. Sydney: NSW DET.

Pesek, D. D., & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education*, *31*(5), 524-540.

Pirie, S., & Kieran, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics, 9(*4) 7-11.

Pirie, S. & Kieran, T. (1994). Beyond metaphor: Formalising in mathematical understanding within constructivist environments*, For the learning of mathematics, 14*(1), 39-43.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking*. (pp. 110-151). New York: Academic Press.

Romberg, T. A. (2000). Changing the teaching and learning of mathematics. *Australian Mathematics Teacher, 56*(4), 6-9.

Sowder, J. (2001). Connecting mathematics education research to practice. In J. Bobis. B. Perry, & M. Mitchelmore (Eds.). *Numeracy and Beyond: Proceedings of 24th Annual Conference of the Mathematics Group of Australasia*, University of Sydney, June-July. (pp. 1-8).

Tickle, B. (2000). Helping children to articulate their thought processes: Part 2: Making things visible. *Square One, 10(*4), 2-4.

Van den Heuvel-Panhuizen, M. (2001). Realistic mathematics education in the Netherlands. In J. Anghileri (Ed.). *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom*. (pp. 49-63). Buckingham: Open University Press.

Wright, V. (2000). The development of numeracy frameworks. *New Zealand Principa,l 15*(4), 4-7.

Yackel, E. (2001). Perspectives on arithmetic from classroom-based research in the United States of America. In J. Anghileri (Ed.). *Principles and practices in arithmetic teaching: Innovative approaches for the primary classroom* (pp. 15-31). Buckingham: Open University Press.

Young-Loveridge, J. M. (1999). The acquisition of numeracy. *Set Research Information for Teachers*, One, No. 12, 1-8.

Young-Loveridge, J. M. (2001). Helping children move beyond counting to part-whole strategies. *Teachers and Curriculum, 5*, 72-78.

Young-Loveridge, J. M. (2002a). Making judgements about children's numeracy strategies. *Set: Research Information for Teachers*, Two, 29-33.

Young-Loveridge, J. M. (2002b). Early childhood numeracy: Building an understanding of part-whole relationships. *Australian Journal of Early Childhood, 27*(4), 36-42.

Zarzycki, P. (2001). In the clutches of algorithms: A view from Poland. *Mathematics Teaching, 174*, 8-11.