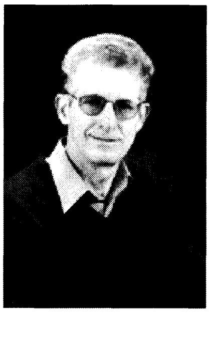
Children's ideas about probability: An action-research investigation

Tania Jones and Jerry Gehrke, School of Education, The University of Waikato.

Tania Jones is a student teacher and Jerry Gehrke is a senior lecturer in the Department of Mathematics, Science and Technology Education.

Children acquire their initial ideas about probability from family and friends in everyday contexts (Barnes, 1998). It is likely that some of their ideas will be incorrect. For example, from experiences playing a game involving tossing a die a child may think that '6' is the 'hardest' number to get, that is, that the probability of getting a '6' is less than the probability of getting any of the other numbers on the die

(Truran, 1995).



Jerry Gehrke

Background

To fulfil the requirements of an assignment in her second-year mathematics education paper, the first author (henceforth just 'the author') carried out a small action research project on the probability ideas of a small group of Year 5 children in a low-decile school.

Small-scale action research is something that teachers can do occasionally as an extension of their everyday teaching. Their initial aim would be to improve their own teaching practice and their understanding of some of the theoretical foundations for their practice (Cohen & Manion, 1989). Additionally, if they tell others about their action research, such as by way of a journal article, they can help other teachers' improve their practice and understanding. So, action research can be a powerful method of professional development for teachers.

Action research can be defined as a "small scale intervention in the functioning of the real world and a close examination of the effects of such intervention" (Cohen & Manion, 1989). Action research is 'situational', that is, concerned with a specific context, 'participatory', that is, the researcher takes part in the research activity, and 'self-evaluative', that is, the researcher continuously evaluates the effects of her actions (Cohen & Manion, 1989). Often action research continues through two or more loops of a plan, act, monitor and evaluate spiral, that is, having done this once, then revise the plan and again act, monitor and evaluate, and so on (Walker, 1985).

Introduction

Intuitive probabilistic thinking is part of our lives. As examples, Mr Kahu decides to take an umbrella because he reckons it might rain, Mr and Mrs Kemp decide to go to the cinema because they think there is a good chance they will enjoy the movie, and Ms Kapoor decides she'll never buy a lotto ticket because she believes it is very unlikely she would ever win more than she would spend. Also, many people use probabilistic reasoning in their work. Examples include weather forecasters, financial planners, medical decision-makers, and persons who set insurance premiums.

Children acquire their initial ideas about probability from family and friends in everyday contexts (Barnes, 1998). It is likely that some of their ideas will be incorrect. For example, from experiences playing a game involving tossing a die a child may think that '6' is the 'hardest' number to get, that is, that the probability of getting a '6' is less than the probability of getting any of the other numbers on the die (Truran, 1995). Children's probability ideas can be investigated or explored in the classroom. This should be done by way of the children carrying out probability experiments and by thinking as clearly and logically about the situations as they can. The teacher's role, of course, is to guide the children in their investigations and explorations, to talk with them and ask questions to help them refine their thinking.

The New Zealand mathematics curriculum document (Ministry of Education, 1992) advocates that teachers should involve their children in problem solving activities within meaningful contexts. Fortunately, there is available to teachers a multitude of contextual problem-solving situations to do with probability. The challenge for teachers is to find out what the children already understand with regard to probability and to design activities that will help them develop better understandings (Barnes, 1998).

The research questions

The author undertook this action research project with the following sequence of linked questions in mind:

- Where in their lives so far have these Year 5 children encountered probability?
- What probability understandings (including misunderstandings) have they constructed from these experiences?
- How can I help them build on their probability experiences and understandings to construct better or further probability understandings?

The research design

The research involved the author as a participant observer in a situation in which she worked with a group of three Year 5 children for two one-hour sessions a week apart.

The first session was mainly an 'exploratory' one. That is, the main aim was for the author to ascertain the probability understandings and language the children already had. This was done by involving the children in two activities and by engaging the children in a discussion related to each activity. Each discussion was guided by focus questions to do with the probability ideas embodied in the activity as well as other possible embodiments of these ideas in other areas of the children's lives. Another aim was to obtain ideas for promising contexts to use to try to help the children further develop their probability understandings and language. This was done by informally talking with the children and asking about their hobbies or other interests. Although 'teaching' in the sense of trying to help the children learn was not a main intention of this session, nor were opportunities to 'teach' avoided if they presented themselves.

The second session, on the other hand, was mainly a 'teaching' session in which the aim was to improve or extend the children's understandings and language, especially their mathematical/numerical understandings and language, to do with probability. The specific objectives were based on the outcomes of the first session.

As soon as possible after each session the author read anything the children had written, reflected on the session, and wrote an account of it.

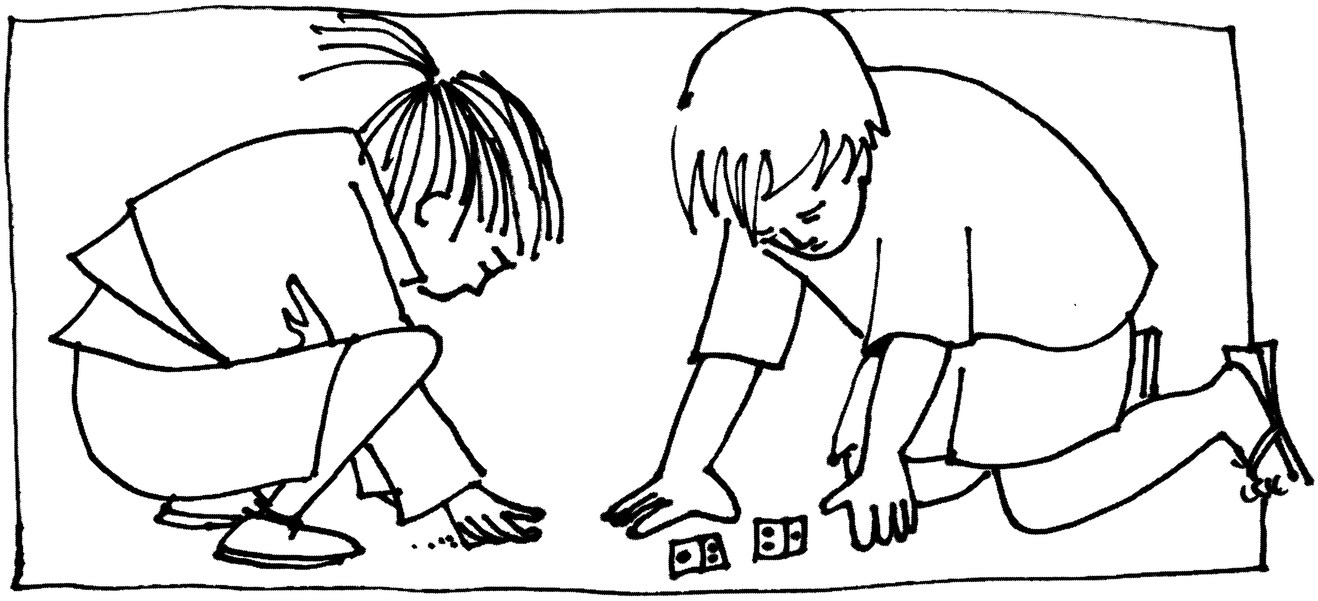
Session 1: Description, results and discussion

The activities used and the ideas/language explored were as follows:

(1) Activity: Play, then analyse, the river crossing game, first using one die; and then using two dice. (The dice are ordinary 'fair' dice, each numbered '1' to '6'.) In the one-die game each player has seven positions, numbered '0' to '6', along one side of a river; and seven 'swimmers' (counters); each player places her swimmers at her positions; for example, she might place one swimmer at position 0, two at 1, none at 2, none at 3, three at 4, one at 5 and none at 6; somebody throws the die; each player who has at least one swimmer at the position number shown on the die has one such swimmer enter the water and cross the river; the first player to get all their seven swimmers across the river wins.

The two-dice game is similar; the position numbers are '1' to '12'; each player has twelve swimmers; if when the dice are thrown the sum of their numbers is, say, 8, then each player who has a swimmer at position '8' may have one such swimmer swim across the river.

Ideas/language:
Chance/likelihood/probability; the probability of throwing any number on a die (for example, 'one chance in six' or 'one out of six' or 'one-sixth' or '1/6'); the relative chances of getting the various possible sums when throwing two dice (for example, there is more chance of getting a



sum of 8 than a sum of 10); (perhaps) the actual probabilities of getting the various possible sums when throwing two dice (for example, the probability of getting a sum of 8 is '5 out of 12' or '5/12'; fair game; (perhaps) fair die; equally likely, not equally likely; strategy.

(2) Activity: Find all the different kinds of McDonalds 'meals', if a meal comprises one drink, one burger and one dessert, and there are 2 kinds of drink, 3 kinds of burger and 2 kinds of dessert from which a meal may be made.

Ideas/language: Combine, combination; all possible combinations; methods used to find all possible combinations; the number of all possible combinations.

For activity (1) the author first explained the rules for the one-die river crossing game and asked the children to play. All chose to put one swimmer at each of their positions. Children took turns to throw the die; the author kept a running record of the results, in case it might be helpful to refer to this later. After about ten throws the author asked the children what they thought the chance was of throwing a '0'. The children took some time to think this through before they realised that it was impossible to throw a '0' and that they should not, then, place a swimmer at position '0'. The children crossed out position '0' and played the game a few times.

The two-dice river crossing game was played immediately afterwards. To start with each child again chose to place one swimmer at each position, and the author again had to ask them a question and allow time for consideration before they realised that it was futile to place a swimmer at position '1'. It soon became evident that the children could not add the two numbers on the dice reasonably quickly. To remove this impediment the author did the adding and called out the sums; also, she kept a tally record of these. The children played the game a couple of times.

Then the author conducted a brief discussion about the river crossing games and noted the following:

- All the children believed that when throwing one die each of the numbers 1 to 6 has the same chance of occurring.
- When throwing two dice one child had a hunch that some sums were more likely to occur than others, but she could not explain why she thought this.
- All the children thought the river crossing game, whether played with one die or two dice, was 'fair', that is, all players had the same chance of winning. They offered other examples of fair and unfair games. They characterised a game as 'unfair' when one player, due to having more experience or skill than another player, is more likely to win than the other player.
- Although once the author had introduced some particular chance word ('fair', 'unfair', 'chance', 'likelihood') the children would sometimes use it, too, they introduced no other chance or probability language themselves. Also, they never used any numerical expressions to attempt to quantify chance.

For activity (2) the author explained the problem in everyday language and asked the children to try to find the answer individually, using pencil and paper to help them think it through. It soon became clear that two of the children had neither sufficient writing skills nor sufficient powers of concentration to investigate this combination activity alone in the remaining time. So, the author decided to help the children tackle the problem together and intervened. She asked them how many meals they thought there might be; they responded '2', '4', and '8'. (There are 12 different meals; so, all the children under-estimated.) Then the author acted as scribe while the children, in turn, explained their thinking. This led to the listing of meals in two columns, one for each of the two kinds of drink. However, after that systematic start, the children seemed to put burger and dessert combinations in the two drink columns in a random manner. Part of the way through the author again asked the children how many meals they thought there would be; two children responded quickly and apparently without much thought; they again under-estimated ('8' and '9'). However, the third child looked carefully at the lists, counted the number of meals listed so far, and then said '12' (the correct answer). He was, though, not able to explain how he came to this answer. Work continued on the list in a random manner until ten meals were found. The author asked the children if they could think of an easier way to work out all the possible combinations. One child indicated she would like to try. Given a piece of paper she constructed a list, formatted in the same two-column way as the first attempt but taking a more systematic approach with the burgers and desserts. Although she did not get to complete this before time ran out, she indicated (correctly) that there should have been two more meals. During this activity all the children showed they could tell when a suggested burger-dessert or dessert-burger was already on the list and so should not be added to it.

Session 2: Description, results and discussion

The author aimed to help the children

- extend their understanding and language to do with chance, for example, to notions of 'poor chance', 'good chance', 'no chance', 'certain';
- begin to move from notions of chance to notions of probability, that is, chance quantified, for example, 'one chance out of three' or 'a one-third chance/probability' or 'a probability of 1/3';
- invent and/or learn some systematic methods, including tree diagrams, for finding all possible combinations.

To help the children achieve these outcomes the author

- introduced a chance line and helped the children plot some examples relevant to their own lives, including the chance they thought they had of winning a play station game in a competition advertised in a local newspaper.
- showed the children how to draw a combinations tree for the McDonald's meals problem from the first session; helped them work together to create their own combinations tree, structurally the same as the meals tree but using a new problem, namely, how many different pants-shirt-shoes outfits Mrs L (their teacher) had if she had 2 different pants (brown, black), 3 different shirts (white, yellow, black) and 2 different pairs of shoes (brown, black) from which to put together her outfits;
- used the Mrs L's outfits context for a discussion about the chance (for example, 'poor chance') and probability (for example, '2-out-of of 12' or '2/12') of Mrs L obtaining different sorts of outfits (for example, an outfit which has black pants and yellow shirt) if she takes and puts on pants, a shirt and a pair of shoes without looking; helped the children plot some of the chances and probabilities on our chance/probability line;
- showed the children a grid for working out all the ways of obtaining each possible sum when throwing two dice and asked them to complete it discussed chances and probabilities of obtaining various of these sums (for example, 'there is a good chance of getting a sum larger than 4'; 'the probability of getting a sum of 9 is 4 out of 12, or 4/12'); had the children plot some of these chances and probabilities on our chance/probability line;
- had the children play the two-dice river crossing game, including explaining their reasons for putting their swimmers at the positions they chose; kept a tally record of the sums obtained; discussed the results, including whether these seemed to be reasonable in light of the analysis of all the ways of obtaining each possible sum and whether we should perhaps throw the dice more times to see if the results then seem 'reasonable'.

Despite the children indicating that they had not come across a chance line before, they understood it relatively quickly and were able to place 'winning a play station game' at a correct position on the line and give some correct reasons for placing it where they did.

When the author explained how to draw the combinations tree for the McDonald's meals, the children indicated that they had not seen such a tree before, and they needed the author's help to interpret it. The author then showed them how the number of possible meals could be found by counting the number of final branches or, alternatively, by multiplying the numbers of drinks, burgers and desserts (that is, 2 x 3 x 2). Then, with only a little help from the author, the children were able to draw their own combinations tree for Mrs L's outfits and find the number of possible outfits by counting the number of final branches.

Now, the children had previously indicated that they had limited knowledge of fractional numbers, understanding little more than halves and quarters, verbally but not numerically. However, within the context of quantifying the chances of Mrs L obtaining various outfits the children understood and used phrases such as 'two out of twelve' and seemed to understand fractional number words such as 'two-twelfths and numerals such as '2/12', when the author said or wrote these, respectively. Within this context, also, the children found it easy to pose their own chance questions, figure out the answer in quantified terms, state the answer verbally, and place it on the chance/probability line.

Examples:

Child 1	"What would be 12 out of 12? What is Mrs L definitely going to wear? Clothes; pants, shoes, tops."
Child 2	"What is the chance of Mrs L wearing something black today? 10 out of 12, a pretty good chance."
Child 3	"What is the chance of her not wearing black? 2 out of 12. Not much chance."

It seems that the children had not come across or used the word 'probability' before. Although they seemed to understand the author's explanation of 'probability' (in terms of using numbers to state chances precisely) and appeared comfortable with her use of the word, none of the children ever used the word themselves.

After a brief explanation and discussion of the grid showing all the ways of obtaining sums when throwing two dice, the children all *seemed* to understand the relative chances of the various sums occurring (for example, '7' is more likely than any other sum, '3' is less likely than '4', and so on). However, upon then setting out to play the two-dice river crossing game again both Child 1 and Child 2 placed one swimmer at each of the 12 positions, just as they'd done in the first session! Child 3 bunched her swimmers around the middle positions, indicating that she did understand the sums' relative chances of occurring. After the author suggested to Child 1 and Child 2 that they might like to consider the grid again, Child 2 moved most of his swimmers to middle positions but still placed one swimmer at position '12', as if to test the theory. Child 1 kept her swimmers where they were. They played the game; Child 3 won.

In discussion afterwards Child 1 said that Child 3 won because she'd placed her swimmers better than he had. In the next playing of the game he bunched most of his swimmers around the middle positions but still insisted on having one swimmer at each of positions '11' and '12'. This time no child put a swimmer at position '1'; when asked 'Why?' they all replied that it was impossible to throw a sum of 1. Again Child 3 won. Child 1 and Child 2 continued to the point where they both had just one swimmer left, at position '12'; finally, on the game's 27th throw, '12' was thrown; both children said they'd place their swimmers differently next time.

Through the sequence of activities the children showed they were beginning to understand that:

- Chances can be described using various words/phrases.
- In any situation there is a range of chances (from 'no chance' to 'certain').
- In some situations chances can be quantified, that is, the chances can be specified by use of numbers.
- Systematic and/or logical ways of recording and thinking about mathematical problems, such as those to do with chance/probability, are helpful and, indeed, necessary.

Edwards and Hensien observed that students are assisted in learning "to quantify chance by relating the numerical expressions, such as "one out of three," to the physical activity that gave rise to those numbers" (Edwards and Hensien, 2000, p. 529). In the author's session the children's physical involvement in the construction of the combinations tree seemed to help them develop a better understanding of the numbers involved than would otherwise have been the case. Also, the children's physical linking of the words and numbers from each activity to positions on a chance/probability line seemed to help them grasp the probabilities involved.

Other researchers have found that sometimes use of contexts either did not succeed in motivating children to participate or caused unexpected misconceptions or 'cheating' in order to obtain the results the children wanted (Taylor and Biddulph, 1994). So, to ensure the children felt the combinations context fitted well enough with the reality with which they were familiar, the author had the children specify the values of the variables in the context, for example, the values 'brown' and 'black' for the variable 'pants'. The children specified 'black' for each variable because they'd noticed their teacher actually did wear a lot of black items of clothing. Subsequently, the result that 10 out of the 12 possible outfits in the context contained something 'black' fitted well with the children's perception of the real situation. The context seemed to belong to them and make sense to them more than might otherwise have been the case, for example, if the author had used a 'dress-the-teddy' context (English, 1992). Also, dressing the teacher rather than themselves precluded comments such as "I wouldn't wear that shirt with those pants".

English maintains that students must "be given the opportunity to discover combinatorial ideas themselves rather than blindly follow rules given to them" (English, 1992, p. 77). However, the author chose to provide the children with a systematic framework for finding all combinations for two reasons, namely, (i) lack of time, and (ii) one child's limited writing skills. Doing so appeared to provide some structure for the children's workings, and they were quickly able to use the framework themselves.

With further learning sessions, using other contexts and activities, these children could continue to develop their understandings and language to do with chance/probability together with their ability to solve problems and take systematic/ logical approaches in doing so. However, it may also be that these children need some special interventionist teaching to improve their basic numeracy and literacy. (Indeed, probably these children should have had such intervention some time ago.

Conclusion

The author found in her 'exploratory' session that despite the prevalence of chance/probability in everyday life in general, these particular children seemed to have had very few encounters with chance/probability and very little knowledge in this area beyond the notion of 'fair'. Despite this, and despite their limited numeracy and literacy, over the course of the 'teaching' session the children seemed to grasp some chance/probability concepts and language relatively quickly, as well as learn how to interpret some fairly complex diagrams and some basic numerical representations.

Several factors seemed to combine to facilitate the children's learning in the second session; these included:

- the teacher knowing the children (at least a little);
- the teacher being able to provide a lot of close attention to the children (due to the low teacher-pupil ratio);
- the teacher working with the children as a social group rather than as isolated individuals, and the children themselves working together cooperatively;
- the teacher using contexts to which the children could easily relate;
- the teacher not rushing the introduction of new language or notation;
- the teacher involving the children in physical activity.

However, it seemed, that with these particular children, the main factor was the following:

- The teacher provided the assistance that was needed to circumvent the children's literacy/numeracy limitations. In particular, she focussed the children's attentions on the central concepts in the activities and avoided having them get caught up in the mechanics of the activities.

References

Barnes, M. (1998). Dealing with misconceptions about probability. *Australian Mathematics Teacher, 54*(1), 17-20.

Cohen, L. & Manion, L. (1989). *Research methods in education* (3rd edition). London: Routledge.

Edwards, T. & Hensien, S. (2000). Using probability experiments to foster discourse. *Teaching Children Mathematics, 6*(8), 524-529.

English, L. (1992). Problem solving with combinations. *Arithmetic Teacher, 40*(2), 72-77.

Ministry of Education. (1992). *Mathematics in the New Zealand Curriculum*. Wellington: Learning Media.

Taylor, M. & Biddulph, F. (1994). 'Context' in probability learning at the primary school level. *SAMEpapers 1994.* Hamilton, N.Z.: Centre for Science, Mathematics and Technology Education Research, University of Waikato.

Truran, K. (1995). *A workshop by chance.* Auckland: New Zealand Association of Mathematics Teachers.

Walker, R. (1985). *Doing research: A handbook for teachers.* Cambridge: Cambridge University Press.