THINKPIECE: MAKING SPACE FOR MATHEMATICS LEARNING TO HAPPEN IN GROUP WORK: IS THIS REALLY POSSIBLE?

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My research in primary mathematics education in England focused on the teaching of calculation strategies following the introduction of the English numeracy strategies (Murphy, 2004, 2011a). My recent arrival in New Zealand as a lecturer and researcher in primary mathematics has given me a fresh perspective in mathematics education, particularly in children's learning in numeracy. This has led me to further review my understanding of different pedagogies and their underlying philosophies.

Policy moves in England and New Zealand, as in other countries such as United States of America and Australia, have promoted the direct instruction of explicit mathematical representations and procedures. Such instruction was intended to improve performance in numeracy by developing a more connected view of mathematics (Ewling, 2011). In direct instruction, representations of number and strategies for carrying out calculations are presented explicitly to children. In New Zealand, the Ministry of Education (2008a) has set out explicit representations of the key ideas in numeracy that children are expected to acquire in a series of professional development booklets.

As with the teachers in England, teachers in New Zealand are accountable for children's progress towards the expected knowledge and understanding as set out in the New Zealand Curriculum and the National Standards. However, there are concerns that direct instruction can result in children acquiring disparate skills and knowledge rather than engaging children in broad mathematical concepts that allow interpretations to evolve (Schleicher & Tamassia, 2000).

Contemporary to the publication of the professional development booklets, Anthony and Walshaw's (2007) review of research-based evidence on effective teaching suggested the need for flexible tasks that engage children in broader mathematical concepts. Studies in New Zealand have promoted children's engagement in discussion and problem solving (Anthony & Walshaw, 2009; Hunter, 2005), and my own research in England has shown how children's engagement in problem solving through discussion in collaborative group work enriched their learning (Murphy, 2013).

These two publications, Anthony and Walshaw (2007) and the New Zealand professional development booklets (Ministry of Education, 2008a) seem to present two main approaches to teaching: one related to direct instruction, and the other related to discussion and collaboration or dialogic pedagogy. My research into teaching numeracy has led me to review philosophical perspectives that underlie different pedagogies in primary mathematics (Murphy, 2011a, 2012) and these two pedagogies would seem to be underpinned by contrasting structuralist and non-structuralist perspectives.

Children's learning: Structuralist and non-structuralist perspectives

From a traditional structuralist perspective, mathematical objects are seen as a system of stable patterns (Lerman, 2001; Sfard, 2001), and children learn mathematics by constructing these stable patterns. This would seem to be the philosophical perspective underlying the direct instruction approach presented in the New Zealand professional development booklets. These booklets present teachers with ways to model ideas in number. For example, place value is modelled through physical manipulatives and pictorial images. Different manipulatives and images model different representations of place value, such as grouping (ten is one group of ten units, hundred is ten groups of ten, and so on) or as symbols written in columns (Hundreds, Tens and Units). To develop a connected view of place value, the teacher models the manipulatives and representations together (Ministry of Education, 2008b, pp. 23–24).

Freudenthal (1983) referred to the teaching of representations and strategies as the passing on of *ready-made* mathematics. Procedures and representations model these ready-made or finished versions of mathematics and, through direct instruction, the teacher leads the children down the steps of a path towards these versions (Ewling, 2011). The children explain their thinking as a sort of *strategy reporting* (Wood & McNeal, 2003) which provides the teacher with a window into the children's minds (Lerman, 2001; Sfard, 2001) to check their construction of these versions.

With a recent social and linguistic turn in the study of mathematics education (Lerman, 2001; Walshaw, 2013), a non-structuralist view of mathematical objects is now proposed. In this case understanding in mathematics is seen as a generative process of meaning making (Wells, 2001). Mathematical objects are not fixed, structured entities that everyone can see and share; they evolve and change as children make meaning within the context of a mathematical problem. The concern is not whether the child can report back a ready-made strategy or representation but whether the child's thinking, in solving a problem, is evolving towards a valid understanding of broader concepts.

This non-structuralist view of mathematics fits with a dialogic pedagogical approach where language (gestures and symbols as well as words) is a mediating tool for learning. Children's learning no longer relates to direct instruction and the prescriptive and explicit modelling of representations and strategies but children are enabled to express and share their own ideas (Alexander, 2004).

The issues in directing teaching or creating a space for learning in group work

These two pedagogies, direct instruction and a dialogic approach, and their underlying philosophies are illustrated by the teaching that happens in group work. In direct instruction, the teacher prescriptively addresses the needs of the children in a group. The children may work individually as the teacher models, observes and coaches. When teaching a group of children from a dialogic approach, the teaching is not prescriptive. The teacher steps aside, and the children select the mathematics to solve the task. The children direct the talk to share meanings amongst themselves in the context of the task. Mathematics learning happens within the space created by the teacher stepping back; within the context of the task and the children's exploration of ideas.

This notion of creating a space for learning raises a possible dilemma. There are certain key mathematical ideas that it would be useful for children to learn, such as the example of place value mentioned above, and children's evolving meaning of these ideas should not be left to chance. Also, mathematics is created from culturally constituted logic and conventions, and ideas cannot be determined entirely subjectively by an individual or by consensus within a group. Evolving meanings have to be valid within the cultural logic and conventions of mathematics.

Even so, direct instruction may not be the way to ensure this dilemma is avoided. Whilst the teacher tells the children "what they need to know and learn" (Ewling, 2011, p. 68), there is limited time and space for children to develop their own pathways or to express their own thoughts. Direct instruction provides discrete strategies and representations, and there is a reliance on children making meaning of the representations and procedures that are presented to them ready-made, and in making connections between the different representations and procedures (Murphy, 2004). In a dialogic approach children are trusted to use their own ideas and arguments and to determine the validity of their solutions. There is an acceptance that their understanding will evolve and change as they take mathematics ideas into different problem-solving situations. In doing so they are making meaning of mathematical ideas in a broader way and becoming critical of their own understanding.

My research in England identified teachers who were grappling with this dilemma. Although the teachers felt they were required to be prescriptive in directing the children's learning, they were not satisfied that the children had time or space to develop their own ideas. During a project to develop collaboration and talk in small group mathematics, the teachers saw how children developed their own paths and thoughts in solving problems. The teachers saw that by stepping back they were creating space for the mathematics learning to happen as the children explored ideas. However there remained a concern that some children might be left to drift in the space created as the teacher stepped back and that they would not be able to explore the key ideas in solving the problem.

Where do we go next?

As noted earlier, the issue is how to create the space for mathematics learning to happen and also to ensure that children's understanding is mediated towards valid solution strategies that will evolve understanding of key ideas in numeracy. From my research, I found that a balance was needed between the definition of mathematical ideas within the context of a task and the teacher's explicitness in explaining the ideas (Murphy, 2011b). The context of a task is provided by the resources used and the organisation of a problem. If the context defines the mathematical ideas too precisely, and if the teacher's explanation is too explicit, then there is little or no space for children's exploration of ideas. On the other hand if the teacher gave little explanation and the task was not well-defined, then the children did not focus on the key idea. Where there was a balance, there was sufficient direction through the context of the task and the teacher's explanation for the children to notice the key idea, but with space for exploration and exchange of meaning.

As Freudenthal (1991) stated, there is "a subtle balance between the freedom of inventing and the force of guiding" (p. 48). So the question arises: How do teachers find this subtle balance between directing and making space for mathematics learning to happen? A quest for us as teachers and researchers together lies in how research can help teachers in making decisions about when to step back to make sufficient space for learning to happen and when to step in to guide and direct.

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