How do we assess mathematics in a constructivist way?

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Introduction

Over recent years the theory of constructivism has become the dominant theory onto which we are expected to build our teaching practice.

Constructivism is a theory of knowing which is based on two main principles. These are:

(a) Knowledge is passively received but actively built up by the cognizing subject.

(b) The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality.
(von Glasersfeld, 1989).

In simpler terms this means that a person learns by "constructing" their own knowledge from the experiences they have in the world around them. These 'constructions' can be thought of as a cluster of inter-linked 'ideas' which help us to adjust, adapt and create our own individual concept of the world.

Our personal view of the world becomes more complex as we construct more knowledge and find answers to the things we do not already know.

When it comes to constructivist teaching, we adapt our teaching practice to fit this theory of learning, so that students may be able to 'construct' their mathematical knowledge from the experiences that we provide for them.

If we teach in a constructivist way we must also learn to assess in a constructivist way as "instruction and assessment are closely linked". "As the forms of mathematical teaching become more diverse – including open-ended investigations, cooperative group activity, and emphasis on thinking and communication – so too must the form of assessment change. No longer can we rely on multiple-choice and short-answer tests to provide all the information on students' mathematical attainments, progress and proficiencies. Changes in curriculum and instruction must be accompanied by equally substantive changes in assessment policies, procedures and instruments" (Stenmark, 1991).

The purpose of constructivist assessment, I believe, is threefold. It has a responsibility to provide:

(a) a learning experience for students

(b) a guide as to future teaching direction for educators

(c) a grade or reporting criteria for parents, administrators etc.

The first two of these *can* happen at the same time if the assessment process is given in such a way that students develop their thinking while explaining what they know. Analysis of these explanations allows the teacher to be able to build future lessons based on these. The third purpose of assessment is more difficult to provide answers for and is still a 'learning field' for teachers and researchers alike.

In this article I intend to examine some ideas for constructivist–type assessment practice and some of the ways in which current researchers have suggested they could be graded or recorded. I will examine the following assessment methods:

1. using performance assessment
2. modifying current testing methods
3. using open-ended questions
4. using problem solving
5. using projects and investigations

Each of these will be considered as separate methods and grading suggestions will be included with them.

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Using Performance Assessment

In the past, we have assessed how well students have remembered knowledge and skills often by timed, written tests and examinations which allow no avenues for students to pursue their own inquiry. Constructivist teaching makes us more aware of the need to assess our students thinking and idea construction. "Increasing attention is being placed on how effectively students can tackle unstructured problems and investigate novel, open-ended situations. The focus of assessment is on how well students have acquired those processes or strategies that guide the choice of appropriate skills and enable students to explore unfamiliar situations" (Swan, 1996).

"A performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing, interviewing and looking at the products to assess what they actually know and can do" (Stenmark, 1991). This assessment can take place during class time and become part of the instructional programme. The task can be written for group or individual work and the teacher can observe each individual's performance while the students develop ideas and build concepts from each other's understandings. Because the teacher is exposed to more of the students' actual thinking than would happen in a conventional test, the teacher gains more insight into student misconceptions or errors and can change instructional processes to overcome these.

The time constraints that are often present in conventional testing are not present to the same extent in performance assessment. The teacher allows the student to develop mathematical thinking at their own pace and in engaging context situations. This learning experience can also enhance motivation for students and empower them in their mathematical experiences.

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In the book 'Mathematics Assessment: Myths, Models, Good Questions and Practical Suggestions' (Jean Stenmark, 1999) there are many examples of sample tasks that could be used by teachers for this type of assessment. The book also provides ideas as to how this performance assessment could be recorded. It suggests that "we can use portfolio presentations, conferences, observation reports, checklists, and descriptive reports as well as grades and test scores". Because lack of time is often a deterrent to many descriptive forms of recording, the book provides samples of a checklist, a collaborative rating form and a self-assessment checklist These may be useful guidance charts for teachers when evaluating student learning. These assessment records have been presented to parents (Stenmark 1991, Wilcox 1998) and found to be well received, as they give examples of <u>real</u> performance by students. Whether they will pass the test of reliability and validity by other academic institutions in the future has yet to be tested.

Modifying our current testing practices

Constraints of time, spent on finding/creating suitable tasks and preparing assessment programmes for them, will often lead teachers to 'take the easy way out' and revert to traditional methods of assessment such as tests which examine knowledge and skills. To avoid this 'opting out' Chappell and Thompson (1999) have suggested that we modify our test questions to help us assess student thinking. By adjusting our test questions with 'why', 'how', 'explain', 'discuss' type questions, following skills questions, we can make them more open-ended. In this way we make routine questions more non-routine and prompt students to respond at a higher conceptual level. "Such communication helps in assessing not only the mathematics that students know but also on their understanding of the mathematics" (Chappell and Thompson, 1999).

Chappell and Thompson provide examples of this type of questioning and samples of student responses to these, so that we can get an insight into how effective this method is at displaying thinking. The act of discussing and explaining thinking and strategies is often a new skill for our students in mathematics and one that we need to encourage in our classrooms. The more they are exposed to this type of assessment the more confident they will become at expressing their ideas and being prepared to explore further.

Because this technique is an adaptation of current testing practices, grading is relatively easy although is does require a judgement call by the teacher in assessing thinking. This is an improvement on past practices and perhaps will act as a stepping stone in our changes to a more constructivist-type assessment.

Using open-ended questions

Making standard test questions into more open-ended ones is not the only way to get our students to explain their thinking. Often open-ended questions can become assessment tasks in their own right. Whether we assess these by a performance assessment method or as a written test, in groups or as individuals, is up to us. An open-ended question is one which allows students to respond in a variety of successful ways. The difficult part in writing them is that they need to be open to solution by students at all levels of understanding. In this way each student can give an answer which fits his/her understanding and the teacher can build on this.

In Billstein's paper 'The Stem Model' (1998) examples are given. Students are asked to write their solutions to open problems by using mathematical language and their best representations to communicate how they solved the problem, the decisions they made as they solved the problem, and the connections they made to other problems, mathematics, or other subjects.

Students' work is assessed according to the following five criteria:

- Problem Solving
- Mathematical Language
- Representations
- Connections
- Presentation.

Ability in these is recorded on a graded assessment sheet which is completed by the teacher as well as the student. "Through self-assessment and teacher-assessment criteria, students learn exactly what is important in good problem solving and communication and can evaluate their own progress in these areas" (Billstein, 1998).

These teacher assessment and self-assessment sheets[7] are clear, concise, user friendly time-wise and provide a useful learning experience for student and teacher alike. If open-ended questions are a method of assessment within a class then grading and reporting are fairly straightforward when using this type of generalized assessment sheet.

In assessing open ended questions Stenmark (1991) suggests that "evaluation of open-ended questions can be analytical with points for various aspects of the response, or holistic, with the reader and evaluator looking at the paper as a whole rather than searching for specific details". This holistic approach could be assessed by using an analytic scoring scale that the author provides.

Using Problem Solving

Problem solving has been separated from open-ended questions, for the purpose of this paper, as it is not necessarily always open-ended. A task like 'designing a chair' is much more open than a calculation that follows particular steps and has a specific answer. If problem solving is going to be

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effective as a learning and assessment experience for student and teacher, it needs to present a perturbation that requires the student to think about the mathematics involved and make decisions about how to construct an answer. It is not acceptable constructivist teaching for students just to answer problems that follow algorithmic method. "Most of the text book and homework 'problems' assigned to students are not problems....but exercises". "Real problem solving confronts students with a difficulty. They know where they are, and where they want to get – but they have no ready means of getting there" (Schoenfeld, 1989).

We also need to be aware that the problems we pose need to be within the real-life experience of the students, eg: problems involving taxes and mortgages may be completely foreign to a student, whereas receiving pocket money or buying at a sale may be more relevant.

The aim of assessing by problem solving is to examine the skills as well as the mathematical thought of our students. In developing an assessment plan we first have to solve the problem ourselves. Only in doing this will we gain knowledge of the crucial elements of the problem and be able to assign any sort of value to their importance. Often at this stage a scoring rubric could be constructed and grades allocated for different mathematical thinking steps. Leitze and Mau (1999) are two writers who have developed a scoring rubric.

In developing this rubric the authors went through a four-stage process:

1) **Understand or formulate the question in a problem**
   As a student works through the problem it can be seen if there is 'true understanding' of what the problem is about.
2) **Select or find the data to solve the problem**
   In this problem the teachers required calculations showing some idea of how many hours, days etc in a calendar year or part year.
3) **Formulate problems and subproblems, and select appropriate solution strategies to pursue**
   The answer also has to show evidence of allowances or variation in these times during their lifetime.
4) **Correctly implement the solution strategy or strategies, and solve sub problems**
   The answer requires that there be evidence of correct mathematical processes and presentation of a correct answer.

Depending on the question, other assessors may wish to put evidence of evaluating the 'reasonableness' of an answer into this rubric as well. These four rubric developing steps (in bold) would be useful to use for most problem-solving questions, as they are quite general.

Leitze and Mau are careful to emphasize that when marking this type of problem it is important that you read the entire solution, before allocating marks, as students are not always forthcoming in their explanations of their problem solving techniques. Sometimes you may have to infer ideas from their answers and other times it may be easier for the student to explain their answering process out loud. Until students get familiar with being expected to explain their process on paper; some answers may be fairly sparse on written context. However, it is worth persevering, as the process of explaining requires the student to reflect on their thinking and focus on the meaning of the problem not just the answer. The more connections they are able to make, the better they become at using that information to help them solve problems. Successful students need to develop a fluidity of thinking that encourages them to consider a wide variety of options and then choose the ones that are appropriate to the situation (Hanselman, 1996).

Using Projects and Investigations

Certain topics, such as statistics, may lend themselves more to a project or investigative approach when being taught. With these topics, continuous assessment of individual or group work can take place while the project or investigation is going on. At the same time, the teacher can observe personal characteristics of students such as creativity, initiative, group-participation, persistence, open-mindedness, etc.

For assessing these the teacher could formulate a fairly open guidance and grading sheet for the students to follow or formulate a progress sheet for student and teacher to record onto as the project progresses. Stenmark (1991) suggests a log sheet onto which students record the date, work done, and any questions that arise. When continuously assessing, the teacher reads and responds to these and writes comments to keep the thinking going. The end product may be a report that the group or individual submits for grading. Meanwhile continuous assessment has given the teacher more insight into progress and thinking and the students have been given perturbations to extend their thinking and project to greater depths.

If the teacher also wants to include a grade for group participation of individual members; this can be done by simply giving the students a written and private grading paper to grade other group members. I have used this technique many times in classes doing group work and have found students are quite prepared to be honest about the contributions of others as long as the results are for the 'teachers eyes only'.

Conclusion

Conventional standardized testing has major shortcomings when we rely on these tests for purposes of ranking, reporting, comparing teachers, courses and schools. Many of these tests emphasize low level skills testing at the expense of conceptual understanding. These tests also offer little assessment of attitudes and behaviors such as persistence, flexibility, creativity and cooperation. Students scores are given for right or wrong answers and little credit is given to strategies, processes or progress of thinking.

As we become more diverse in our forms of instruction we must also become more diverse in our forms of assessment. "Assessments embedded in instruction are important sources of information for instructional decisions made by teachers and other members of the educational community. Authentic assessment tasks highlight the usefulness of mathematical thinking and bridge the gap between school and real mathematics to carry out investigations, we can make valid judgements about their achievement" (Stenmark, 1991). We are aware that "documentation

> of assessment is important in connecting classwork to external evaluation" (Stenmark, 1991). However, this should not be the driving force for our assessment. Billstein's (1998) 'STEM model' (a six year mathematical concepts module programme) advocates that:

- Assessment should have the improvement of learning as its primary goal
- Documenting student's achievement should be an integral part of the instructional process, not an add on to it
- Assessment should include the active participation of students in open problems
- Assessment should reflect real-world applications
- Assessment should permit the full use of technology
- Assessment should use a variety of methods".

It is by examining research projects such as this that educators can come to a clearer understanding of how assessment techniques could be adapted to fit alongside our constructivist teaching. In this paper I have presented several approaches to assessment and suggested ideas as to how they can be graded and recorded.

There is still a need for more research into constructivist-type assessment techniques and we cannot expect results in a short amount of time. Assessment is an on-going learning process itself. It is only as we try techniques with our students, reflect on these and adapt them for future assessment will we find a range of assessment techniques that 'fit' with our concepts of how we can assess better in different topic areas. Meanwhile we can use current research to guide us in this process.

We must remember that new forms of assessment are not goals in and of themselves. The major reason for diversifying mathematics assessment is the value that the diversification has as a tool for the improvement of both teaching and students' mathematics learning (Stenmark, 1991).

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