Writing in the secondary mathematics classroom

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Introduction

New Zealand at present is working through some major educational reforms. At the secondary school level these reforms are asking teachers to question both how they teach and how they assess their students. In particular *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) promotes a move away from the traditional 'this is what you need to know and this is how you do it' style of teaching to a facilitative approach in which students are encouraged to construct in their own mathematical knowledge. To further develop this interactive mode of learning Ehrich (1994) suggests that teachers must begin to look for a variety of learning tools that allow students to engage actively in the construction process. One such tool is writing.

In this article I report how students can build mathematical understanding through writing, and the benefits and constraints of such activity. To do so I draw upon relevant research studies and other literature, and also upon experiences gained with some of my own classes.

Mathematical learning through writing

Students perhaps do more writing in mathematics than any other subject but typically the writing involves mathematics symbols, not verbal (Miller, 1993). Most people identify writing with languages and in the past, traditional writing such as letters to friends, journals and expository essays have been kept separate from the mathematics curriculum. Today, however, teachers are being urged to move beyond using teaching methods involving traditional 'chalk and talk', rote learning and graded exercises to use more interactive, student-centred and problem-solving techniques. In part, the drive towards using writing as one of these new tools comes from the current mathematics curriculum. For example, the curriculum (Ministry of Education, 1992) advises that students should

write and solve story problems	*p.41*
record, then talk about ...	*p.81*
write and use equations of ...	*p.122*
write and tell number stories ...	*p.137*

In short, the curriculum actively promotes writing as a technique that can be used to help develop and consolidate new concepts for students.

Researchers such as Burns (1995, Ehrich (1994), Miller (1993) and Mousley and Marks (1991) also believe that writing in mathematics is a beneficial interactive learning tool. For instance, Miller (1993: 51) has observed that writing activities, "develop students' abilities to read, define and hypothesise; inculcate methods of problem solving; assist with the construction of knowledge; recognise attitudes; and promote individual teacher-student interaction." In other words writing about mathematics helps students to become actively involved in the thinking and learning process. Writing encourages students to examine their ideas and reflect on what they have learned. It helps them to clarify their thoughts, to make connections, and to deepen or extend their understanding of new concepts.

Forms of writing

There are two main forms of writing in mathematics. The first is **routine writing** and recording. Routine writing, according to Waters and Montgomery (1993), is used to reinforce a defined step-by-step arithmetic procedure (such as the long division algorithm), to reproduce what has been taught, or to learn mathematical definitions. Types of writing which fall into this category are summaries, translations, definitions, reports, labels, instructions and notes (Richards, 1990).

The second form is **explorative writing.** Waters and Montgomery (1993) consider that explorative writing is used to explore ideas and create new understanding. Students using this form of writing may be writing about their own method used to solve mathematical problems, their personal response to certain concepts and problems, or itemising what they do and do not know about a particular area of mathematics. Evaluations, descriptions, predictions and arguments fall into this second category (Richards, 1993).

Ways that students learn through writing

A class where explorative writing is taking place is a class where students are actively learning by constructing their own mathematical knowledge. Ehrich (1994) discusses four distinct ways that students are able to learn through writing. The first she calls **cognitive dissonance**. Cognitive dissonance is what students experience when they realise that what they thought was true can now be challenged by some new information. When learners experience this conflict they feel uneasy and want to ease their perturbations. The use of writing during this time increases the likelihood of the learner reflecting on past ideas, clarifying current thought and making links between old and new concepts. Once students' perturbations have gone and the new information ha been assimilated into existing mental structures, or the structures themselves have been altered, then learning has occurred (Green, 1995).

In terms of my own teaching I often see students trying to resolve cognitive

conflicts. For example, one of the the ways that I like to introduce negative numbers to my third form class is to ask them to write down a definition for 'number' and to "follow this definition with various examples". Frequently the students give answers such as:

Numbers are things you use to count with, like 1, 2, 3 ...
1 is a number
Something between 0 upwards
A million, million billion

and so on with lots of examples of the biggest numbers they can think of. Then I follow my original question with: "The temperature today is 10° Celsius but it suddenly begins to snow and the temperature drops by 15°. Calculate the new temperature and explain how you know." Typical student responses are:

0° because it is the lowest number
5° because 15 minus 10 is 5
5 ° below freezing because 0 is freezing

We continue to discuss the problem and find that the answer is negative 5 because numbers exist below zero. Many students are receiving this information for the first time. I show them a number line between -20 and +20 and I can see by the expressions on their faces that many are challenged by this new concept. To help consolidate the learning I ask the students to write down any other real life examples they can think of which involve negative numbers. Answers such as water levels, building floor levels, and money are usually readily communicated . It seems that only when the students have been able to either assimilate the new information into their existing mental schemes, or have altered their schemes to accommodate it, that the learning of this particular concept is reasonably complete.

A second way that students are able to learn through writing is by **affirmation** (Ehrich, 1994). In her masters thesis *Cognitive Process Writing in Mathematics* (Ehrich, 1991) Ehrich showed that writing allowed students to affirm and strengthen their understanding of a particular piece of mathematics even after they had arrived at a solution. She concluded that through writing students were able to confirm their ideas and beliefs about a particular mathematical idea or skill, and thus evaluate the accuracy of their own thoughts.

I decided to explore this affirmation idea with my Form 5 class. We were in the process of studying quadratic equations and needed to find general rules for them. I modelled the solution to a particular example in class and gave another for them to complete for homework. I said that at the end of their attempt at a solution they had to write a sentence explaining how they worked out a solution, or explaining why they could not complete the problem. The results were rather interesting. A few students wrote statements such as:

Finished; it's easy.

Here is the answer; no problems.

So the finished form is, put in the simplest form, and finished. Simple.

These students tended to be the International fee-paying students. They had completed the task, reflected on their solution, know it was correct, and were able to affirm their success. Others in the class, however, were not able to do this. Rather, their writings highlighted the particular areas of weakness that they had in understanding the concept. Nevertheless, I believe that writing was very important for these students too; rather than just stopping when they could not complete the task they had to analyse why they could not complete it. Examples of their writing were:

I couldn't solve this because I realised that I couldn't work out the 1st and 2nd difference.

Stuck here [arrow point to position]. I do not know how to write the new equation.

This is where I got confused and can't keep going so I need to work on more work like this to understand, e.g page numbers in the book...

I can do it, but not without my notes.

As a teacher these comments were very helpful because they directed me to the students' difficulties and we were able to work together to solve them. What I had not expected, however, were students who were successful in solving the problem but who were not confident with the concepts because they had used the worked example as a guide and could not do another problem without it. It made me think very hard about how I was going to help them. Should I just tell them to memorise the algorithm, or to try a lot more problems, or should I say, "Don't worry too much because it does not come up that often in School Certificate"? In fact I chose my second possibility; I hoped that if they did more problems they would come to terms with the algorithm and feel more confident about their abilities. Overall I found this exploration very satisfying because I had identified another teaching approach to help students clarify their thoughts and, in so doing, another technique to communicate with the students.

Ehrich's (1994) third way that students can learn through writing is by **exploring**. Exploratory writing is defined by Waters and Montgomery (1993) and Ehrich (1994) as what is done when learners use writing as a tool to explore ideas, create new understandings and solve problems, even though they may not have a route to the solution in their mind when they begin. Waters and Montgomery say that keeping a mathematics log is a good example of explorative writing. a log is essentially a student's journal of work done, and will include comments about problems, trials and findings along the way.

Through explorative writing learners can also experience spontaneous discovery. Ehrich (1994) calls this fourth way that students can learn mathematics

through writing the **Aha mode**. One such example was found in a student's journal where the student was attempting to define the term 'consecutive integer'. She wrote, "I have no idea" but then proceeded to list 10 consecutive integers. Her definition concluded with this suggested a sudden change when she went from being unsure about her answer to being certain she was right.

Again, in terms of my own teaching I admit that I have not used these styles of writing in mathematics to any great extent. I have seen students go "Aha" when their thoughts and understanding about particular concepts have finally clicked, but I have not achieved this through the medium of writing.

Benefits and constraints

Writing in mathematics has its benefits, but there are also constraints. A student who is able to write in mathematics is better able to communicate with the teacher, and the teacher with the student, independently of others in the class. Writing helps students to summarise the numerous information covered within particular units and, as indicated above, gives students time to reflect on particular learning sessions. In this way writing helps students clarify their thoughts, make connections with prior knowledge, and explore or consolidate new ideas. Writing is also beneficial because it can help students focus on the task at hand. Further, it is a medium by which they can express their feelings about mathematics and the problems which they encounter in the learning process. For the teacher, being able to read about students' difficulties is of great value as misunderstandings can be pinpointed.

Constraints, however, do exist. Previous mathematics learning experiences mean that many students do not perceive that writing should be part of work in mathematics classes. Mousley and Marks (1991) believe that for students to benefit from writing in mathematics they need to be taught a variety of writing techniques, or they will merely recount what happened in class. A further problem with writing is that it can take time. The usual structure of 50 minute periods, together with the large amount of content material required for examinations, means that many teachers may feel there is little time to include writing in their mathematics programme. Further, the reading and marking that this sort of activity can entail may also be a deterrent for teachers who have been trained primarily in mathematical content and may not be experienced in analysing students' writings. In my own case, whilst I now believe that writing is an important tool in my collecting of teaching techniques, I find it challenging to know what to do with some students' responses, for example, those 5th formers who were able to solve the quadratic patter but who wrote about lacking confidence to do others.

Perhaps for writing to become an integral teaching technique and learning tool in secondary mathematics, teacher professional development is needed that helps

teachers recognise when it can be employed, and how the information gained can be used to enhance the student's learning. Not to include writing as a learning tool is to deny our students the potential of an exciting new learning technique. Despite the constraints, I shall continue with it myself.

References

Burns, M. (1995) *Writing in maths class: A resource for Grades 2-8.* New York: Maths Solutions Publications.

Ehrich, P. (1991) Cognitive process writing in mathematics: A middle school study. Cited in Ehrich, P. (1994) Writing and cognition: Implications for mathematical instruction. In D. Buerk (ed) *Empowering students by promoting active learning in mathematics.* Reston, Virginia: NCTM.

Ehrich, P. (1994) Writing and cognition: Implications for mathematical instruction. In D. Buerk (ed) *empowering students by promoting active learning in mathematics.* Reston, Virginia: NCTM.

Greenes, C. (1995) Mathematical learning and knowing: A cognitive process. *Journal of Education,* 177, 1, 85-106.

Mousley, J. and Marks, G. (1991) *Discourse in mathematics.* Geelong: Deakin University Press.

Miller, L. D. (1993) Writing in mathematics classes. In B. Fraser (ed) *Research implications for science and mathematics teachers.* Perth: Curtin University of Technology.

Ministry of Education (1992) *Mathematics in the New Zealand Curriculum.* Wellington: Learning Media.

Richards, L. (1990) Measuring things in words: Language for learning mathematics. *Language Arts,* 67, 1, 14-25.

Waters, M. and Montgomery, P. (1993) Writing in mathematics: Multiplying the learning. In M. Stephens, A. Woody, D. Clarke and J. Izard (eds) *Communicating mathematics: Perspectives from classroom practice and current research.* Hawthorn, Victoria: ACER.