## Volume 21 Issue 1, 2021

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Sashi Sharma,Shweta Sharma, Phil Doyle, Louis Marcelo, \& Daniel Kumar
Editor: Kerry Earl Rinehart
To cite this article: SharmaS., Sharma, S., Doyle, P., Marcelo, L., \& Kumar, D. (2021). Exploring probability concepts in a game context. Teachers and Curriculum, 21(1), 59-70. https:doi.org/10.15663/tandc.v21i1.359

To link to this volume: https://doi.org/10.15663/tandc.v21i1

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# EXPLORING PROBABILITY CONCEPTS IN A GAME CONTEXT 

SASHI SHARMA, SHWETA SHARMA ${ }^{1}$<br>PHIL DOYLE, LOUIS MARCELO AND DANIEL KUMAR ${ }^{2}$

The University of Waikato ${ }^{1}$ \& De La Salle College ${ }^{2}$
New Zealand


#### Abstract

Learning about probability can pose difficulties for students at all levels. Performing probability experiments using games can encourage students to develop understandings of probability grounded in real events. In this reflective paper, we explore the thinking of a group of students and teachers as they reasoned about experimental and theoretical probabilities in a game context. We designed a probability lesson based on the game LuLu (McCoy et al., 2007). In this article we share the activity and describe the kinds of explorations that can be facilitated in any secondary school mathematics classroom. We were particularly interested in investigating whether students could construct a bidirectional link between experimental probability and theoretical probability. Overall, the lesson enabled students to gain hands-on experience in data collection and analysis and better comprehend affordances of culturally diverse games.


## Keywords

Experimental probability; Theoretical probability; Chance games; secondary school students; culturally responsive teaching

## Introduction

Probability straddles a number of learning areas (mathematics, sports, physics, economics and sciences) because of its wide range of applicability (Gal, 2005; Leavy \& Hourigan, 2014). Moreover, the language of probability pervades almost everything we do (Groth et al., 2016). In recognition of the importance of probability in both school and out of school settings, there has been a movement in many countries to include probability at every level in the mathematics curricula (Batanero et al., 2016; Leavy \& Hourigan, 2014; Watson, 2006). In schools, two aspects of probability need to be considered, the theoretical and the empirical. A theoretical approach
is obtained by the fraction of outcomes favourable to this event in the sample space; this makes use of an implicit assumption of equal likelihood of all single outcomes of the sample space. It is an a priori approach to probability in that it allows calculations of probabilities before any trial is made. (Borovenik et al., 1991, p. 41)

For instance, a six-sided die with different numbers on sides is rolled once and students are asked to determine the probability of the event 'even numbers'. In this approach, students will determine the theoretical probability by considering the sample space: counting the number of sides with even numbers (i.e., three even numbers) and the overall number of sides (six sides) and determine the ratio of both values as $\mathrm{P}($ even $)=3 / 6=0.5$. However, this specific approach is only applicable under the assumptions that the die is fair and each of its sides has the same surface area and thus the same likelihood to be facing up (1/6).
For the empirical or experimental approach, probability of an event, an experiment is conducted repeatedly and the results are noted. By determining the relative frequency of a long series of repetitions, the probability is estimated. The value of this approach in schools is especially apparent when theoretical considerations are impossible, for instance for very irregular dice. The experimental and theoretical probabilities are closely related and complementary: "The relationship between the two
concepts results from the fact that for a given event, experimental probability will more closely approximate theoretical probability as the number of trials increases" (Jones et al., 1999, p. 148). Hence these two approaches should not be separated if we want students to develop a good understanding of probability and apply it in practical situations (Chaput et al., 2011). School students need opportunities to explore and contrast the theoretical and empirical/experimental approaches to probability as advocated by Steinbring (1991).

In New Zealand, by the end of primary school (Year 8), students are expected to describe probabilities using fractions, conduct chance experiments with both small and large numbers of trials using appropriate digital technologies, and compare observed frequencies across experiments with expected frequencies. Moreover, the use of meaningful contexts and drawing on students' experiences and understandings is recommended for enhancing students' understanding of probability concepts (Ministry of Education, 2007). Like in many other learning areas, learning probability is well known to pose obstacles when children bring in their everyday experiences and prior conceptions which are not always in line with the mathematical concepts concerning chance and probability. While there is a considerable and rich literature on students' intuitions and misconceptions in probability, less attention has been paid to the development of students' probabilistic thinking in the classroom (Sharma, 2014).
An intriguing recommendation for teaching probability is to use culturally diverse games to promote students' understanding of probability (Carlton \& Mortlock, 2005; Koparan, 2019). It is argued that a probability lesson embedded in a cultural context can enable students to reflect on the connections between content (probability) and context (culture) and as a result broaden students' perceptions of mathematics and statistics (Averill et al., 2009). Koparan (2019) states that game-based learning can increase student motivation and can be used to evaluate and discuss probability knowledge. Culturally diverse games for probability exploration can be used in statistics classrooms because such activities not only provide a "legitimate case of straightforward mapping of situations onto probabilistic structures" (Greer \& Mukhopadhyay, 2005, p. 316) but also allow for simulations using both cultural artefacts and technological tools. In addition, cultural games will help sustain student interest and motivation and help teachers highlight the significance of the role of culture and context in a multicultural statistics classroom (Averill et al., 2009). Recognising a need to investigate how students' learning of probability can be supported by the affordances of culturally diverse games, in our collaborative study, games were used to explore the basic probability knowledge of Year 9 students and to enable discussions around mathematical ideas in a culturally responsive environment.

## Research design and data collection methods

To conceptualise our study, we drew on design-based research theory (Cobb \& McClain, 2004). Design research is a cyclic process with action and critical reflection taking place in turn (Cobb \& McClain, 2004; Nilsson, 2013). There are benefits for both teachers and researcher when undertaking a design research partnership: the research plan can be flexible and adaptable to unforeseen effects or constraints (Nilsson, 2013). Further, all participants are equal partners in the research process with no hierarchy existing between researchers and practitioners (Hipkins, 2014; Kieran et al., 2013).

The following inter-related research questions guided our study:

1. How do multilingual Year 9 students negotiate communication in small-group and whole-class settings when working on probability tasks?
2. How might teachers draw on home languages, contexts and cultural experiences to enhance the probability understanding of multilingual students?

The study itself involved cycles of three phases: a preparation and design phase, a teaching experiment phase, and a retrospective analysis phase. Teachers were involved in the whole research process: posing questions, collecting data, drawing conclusions, writing reports and dissemination of findings (see Sharma, 2017).

- Phase 1: Preparation and design for the teaching experiment. This phase began with a discussion of research findings on probability and language challenges and culturally responsive pedagogical strategies for English Language Learners. The research team (three teachers and a researcher) proposed a sequence of probability ideas, using games, language skills, knowledge and attitudes that they anticipated students would construct as they participated in teaching and learning activities.
- Phase 2: Teaching experiment. The teaching took place as part of regular classroom statistics teaching in two largely Pasifika student dominated Year 9 classes. As part of the learning activities, students generated experimental data and then worked together to determine the experimental and theoretical probabilities of possible outcomes. During the teaching experiment, audio recordings of group discussions were made. In addition, copies of students' written work were kept. Each teacher also kept a logbook of specific events that took place during the data collection period. Logbook entries helped teachers identify and keep track of which strategies seem to work well for students and which ones were less successful.
- Phase 3: Retrospective analysis. The three teachers and researcher performed a retrospective analysis together after each lesson to reflect on and refine the lesson plans while the teaching experiment was in progress. The updated lesson plans were used for teaching future lessons. In addition, the team analysed the whole unit on completion of the teaching experiment cycle.
This research was approved by the University of Waikato Division of Education Ethics Committee.


## Findings

This section focuses on data gathered from the first cycle of the teaching experiment. Each lesson followed a common format. Students learned about the game, including the history and background as well as instructions for playing. Next the teachers demonstrated the game to the class. Then students were placed in small groups of two to four, given appropriate game materials and instructed to play. Students were introduced to probability concepts related to the game, either as an integral part of the strategy or as an experiment where data were collected as the game was played. Students collected and analysed the data and reported their results on the group worksheets as both short answers and longer explanations.

## Posing a problem

We engaged the students by presenting the game scenario below. The students were asked to read through the LuLu game (Naresh et al., 2014) description and make sure they understood what was required. We had to clarify what 'sum' and 'cumulative' meant from probability and mathematical perspectives.

Scenario: Lulu is a Hawaiian game. The literal meaning of the word is 'to shake'. The game involves the use of four glass stones, as shown in Figure 1.


Figure 1: Glass stones showing dots on face-up.

The stones are thrown at random on the table, and the throws are named as follows:

- Hu-li la-lo-all stones are face-down.
- Hu-ka-hi hu-i i-lu-na-one stone is face-up.
- E-lu-a hu-li i-lu-na-two stones are face-up.
- E-ko-lu hu-li i-lu-na-three stones are face-up.
- E-ha hu-li i-lu-na-all four stones are face-up.

The game can be played in groups or pairs. Each player shakes the stones in both hands and then tosses them on the table; dots on the fall face-up are counted for points. The player with the highest score wins the game.

## Teachers modelling the game

Teachers helped students to attend to the probability vocabulary in relation to the LuLu game: experiment (playing the game of LuLu), trial (one toss of the stones), outcome (the way the stones landed), experimental data (scores of individual players), experimental probability (probability of the collected data). Teachers used student feedback to write definitions on the white board. Two teachers played a round of the game described above. Teachers tried to read the outcome using Hawaiian text; for example, they said E-ha hu-li i-lu-na when all sides of the glass stones were up. They discussed the rules of the game and the need to systematically record the data. Teachers played the game five times and recorded the results in a frequency table (as shown in Table 1).

Table 1: $\quad$ Frequency Table

|  | Round 1 | Round 2 | Round 3 | Round 4 | Round 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Teacher 1 | 0 | 3 | 2 | 10 | 1 |
| Teacher 2 | 6 | 2 | 0 | 4 | 9 |

Teacher modelling helped students get familiar with the rules of the game and the ways to record their data. When asked what scores were more likely to occur, some students stated that there were four equally likely outcomes: $1,2,3$, and 4 . After teacher modelling students realised that there were more than four scores including $0,5,6,7,8,9,10$.

## Experimental data collection and analysis

After teacher modelling, the students were asked to play the game in groups of three and record the data. Table 2 shows data from Group A. Group A recorded cumulative data to guess the winner. After eight rounds, they reported that Player 2 won the game with 48 points.

Table 2: Cumulative Data Collected by Group A

|  | Player 1 | Player 2 | Player 3 |
| :---: | :---: | :---: | :---: |
| Round 1 | $7+$ | $3+$ | $7+$ |
| Round 2 | 10 | 9 | 1 |
| Cumulative Score: Rounds 1 and 2 | $17+$ | 12+ | 8+ |
| Round 3 | 5 | 10 | 3 |
| Cumulative Score: Rounds 1 to 3 | $21+$ | $22+$ | $11+$ |
| Round 4 | 3 | 6 | 2 |
| Cumulative Score: Rounds 1 to 4 | $24+$ | 28 | 13 |
| Round 5 | 5 | 4 | 0 |
| Cumulative Score: Rounds 1 to 5 | $29+$ | $34+$ | 13+ |
| Round 6 | 3 | 4 | 5 |
| Cumulative Score: Rounds 1 to 6 | $32+$ | $35+$ | $18+$ |
| Round 7 | 7 | 6 | 7 |
| Cumulative Score: Rounds 1-7 | $39+$ | $41+$ | $25+$ |
| Round 8 | 1 | 7 | 10 |
| Cumulative score: Rounds 1-8 | 40 | 48 | 35 |

The group was later asked to record their data using a frequency table (as shown below) as they played the game for 20 rounds.

Table 3: Data Recorded by Group A Using Frequency Table

| Score | Frequency | Experimental Probability |
| :---: | :---: | :---: |
| 0 | III | 3/20 |
| 1 | II | 2/20 |
| 2 | I | 1/20 |
| 3 | IIII | 4/20 |
| 4 | II | 2/20 |
| 5 | I | 1/20 |
| 6 | I | 1/20 |
| 7 | I | 1/20 |
| 8 | I | 1/20 |
| 9 | II | 2/20 |
| 10 | I | 1/20 |

Based on the data, the students were asked the following questions:

1. What are the possible outcomes for a turn?
2. Are these scores equally likely? Explain your thinking.
3. Calculate the probability for each score?

The discussion led to students realising that although 0 through 10 are possible, the likelihood of some scores (e.g., 3,4 ) are more than the others $(1,10)$.

## Theoretical exploration

Once students had a good understanding of the dynamics of the game, the teachers then posed the following probability questions:

1. What are all the possible outcomes in one toss?
2. How many different ways can each score be obtained?
3. What is the theoretical probability of each score?

In groups, students analysed the game to theoretically determine the probability of each outcome. They used their own methods for listing the possibilities. Most groups enumerated the sample space in systematic ways such as lists (see Figure 2), diagrams (see Figure 3) and tables (see Table 4).

```
4-3-2-1; 4-3-2-0; 4-3-0-1; 4-3-O-0; 4-0-2-1; 4-0-2-0; 4-0-0-1; 4-OO0; 0-3-2-1; 0-3-2-0; 0-3-0-1; 0-
3-O0; OO2-1; 0-0-2-0; 0-0-0-1; 0-0-0-0.
```

Figure 2: Work Sample of one student showing possible outcomes using list.


Figure 3: Work sample of one student showing possible outcomes using diagram.
Table 4: Work Sample of One Student Showing Possible Outcomes Using Table

| Possible outcomes for a turn | Score |
| :---: | :---: |
| 1, 2, 4 | 7 |
| 1,3,4 | 8 |
| 1, 2, 3 | 6 |
| 1, 2, 3, 4 | 10 |
| 2, 3, 4 | 9 |
| 2, 3 | 5 |
| 2, 4 | 6 |
| 1 | 1 |
| 1, 3 | 4 |
| 1, 4 | 5 |
| 2 | 2 |
| 3, 4 | 7 |
| 3 | 3 |
| 1, 2 | 3 |
| 4 | 4 |

Students identified 15 possible outcomes (as shown in the Table 4). They did not include the score 0 , obtained when all glass stones faced down. This shows that some students may not include 0 as one of the possible outcomes. The reasoning behind this could be explored in future research.
The diagram below (see Figure 4) helped students to visualise all 16 possible outcomes, each of the four stones may be up or down. They also noticed that the values in the middle of the distributions are more frequent than others because there are more ways to get them.

| Score | Hu-li la-lo (All face-down) | Hu-ka-hi hu-i i-lu-na (One face-up) | E-lu-a hu-li i-lu-na (Two facc-up) | E-ko-lu hu-li i-lu-na (Three face-up) | E-ha hu-li i-lu-na (Four facc-up) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 OO |  |  |  |  |
| 1 |  | $\bullet \bigcirc \bigcirc$ |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  | $00 \because$ | $\bullet \bullet 00$ |  |  |
| 4 |  | $00: 8$ | $\because \because \bigcirc$ |  |  |
| 5 |  |  |  |  |  |
| 6 |  |  | $0 \cdot 0$ | $\because \because$ |  |
| 7 |  |  | $\bigcirc \because: 8$ | $\because \because \quad \because$ |  |
| 8 |  |  |  | $\bigcirc \bigcirc$ |  |
| 9 |  |  |  | $\bigcirc \because:$ |  |
| 10 |  |  |  |  |  |

Source for Hawaiian names: http://gamesmuscum.uwaterloo.ca/Archives/Culin/Hawaii1899/games/ztoncdice.html

Figure 4: A chart showing all possible outcomes for the game LuLu.

Eleven scores are possible: $10,9,8,7,6,5,4,3,2,1,0$. The scores $10,9,8,2,1,0$ are scored once each, and $7,6,5,4,3$ are scored twice each. Also, the students observed that some scores can be obtained in different ways, whereas other scores can occur in only one way. In other words, the scores of $0-10$ are not equally likely to occur.

Once the students had completed sample space diagrams, the students were asked to calculate theoretical probability for each score. For example, $\mathrm{P}(1)=1 / 16$. Students were asked to note if there was any difference between their experimental and theoretical probabilities and explain the difference. The students noticed discrepancies between the theoretical and the experimental probabilities and attributed this to the smaller number of trials. This observation created an opportunity for teachers to talk about the notion of law of large numbers. We suggest that the data from all groups can be collated (as shown in Table 5) to explore how variation between theoretical and experimental probabilities decreases as the number of trials increases.

Table 5: Collecting Experimental Data

| No of trials Score | Group data | Group experimental probability | Class data | Class experimental probability |
| :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |

## Mathematics involved

'LuLu' is suitable for students in Years 7 to 10. In the game, students are put in the situation of considering the likelihood of getting a cumulative score of 50. Students may interpret the phrase closest to 50 in many ways: getting a score in the range from 48 through 50 , obtaining any score between $50-$ 52 or getting an exact score of 50 . In a group that decides that the winner cannot exceed a score of 50, players may decide to skip their turn as they reach a score closer to 50 . For example, a player whose score is 47 might decide to stop since the chances of obtaining a score greater than a three is much higher than that of either a score of three or less than three. They make judgements and have to choose to take risks or play it safe in order to accumulate as many points as possible. The element of surprise highlights the experience of randomness of the outcomes of throwing four stones. Students have to think about a game strategy and whether to keep playing (thereby taking a risk of a total of three being rolled on the stones) or to miss a turn (thus playing it safe, and retaining the points accumulated TO DATE). Students are continually making comparisons of experimental probability and theoretical probability.
This game may also help students appreciate the concept of independence of chance events. That means that each throw is independent of the previous throw. The probability of getting a two is one in 16 each time, no matter what happened before, even if a two has not occurred for 10 previous throws. Students can correctly list 16 equally likely outcomes and notice that the probabilities of obtaining the 11 scores $(0-10)$ are not all the same. Having students play the game first helps them see why this is the case. The students may notice discrepancies between the theoretical and the experimental probabilities, which can be attributed to the smaller number of trials $(\mathrm{n}=25)$. This observation can be a great opportunity to introduce the notion of the law of large numbers (as stated earlier) according to which, for sufficiently large values of n , the experimental probabilities will more closely reflect the theoretical values.

Simulation can also be used for comparing experimental probability with theoretical probability. In future, we would also like to extend the LuLu game using the challenge version. In the challenge version, a turn consists of two tosses. On the first toss, if all four stones fall face-up, the player scores 10 points and then tosses all the stones again. If all four stones do not fall face-up on the first toss, the dots of the face-up pieces are counted and only the face-down pieces are tossed a second time. In either scenario, a player's score is determined by adding the dots showing on the first toss to those from the second toss.

Students can be asked to repeat the procedures for gathering and recording experimental data, using different representations. Students could use the LuLu applet at http://mathandcomputing.com/lulu/ to gather experimental data within small groups. The challenge version of the game can help students to develop understanding of conditional probability. The challenge version may focus on the following key questions:

1. What scores are possible for a turn?
2. How many ways can a score be obtained?
3. What are the possible outcomes for a turn? Make a diagram/chart.
4. Calculate and compare experimental and theoretical probability for each score.

## Reflections and implications

The results of our preliminary work with students collecting and analysing data are encouraging. Students got quickly engaged with the task, approaching it initially as an act of winning. With prompting, the activity evolved into an act of constructing insights into theoretical and experimental status of probabilities. Although we had planned to use three cultural games in our teaching, due to time constraints we only used the LuLu game. The game of LuLu not only provides an interesting context for discussing probability concepts but also requires students to think deeply about the notions of sample space, representations for sample spaces, and co-ordinating experimental and theoretical probability.
In the next cycle, we plan to use more games to make probability teaching more engaging for students. We will also ask students to individually think about whether the game is fair if, in a game, Player A wins if they get even scores, and Player B wins if they get odd scores. Also, we will ask them to write down their predictions and explanations in their books. Students could use words and diagrams to explain their thinking. As they work on the task, we will listen to their reasoning carefully and note misunderstandings that may arise for later discussion with the whole class. We believe that writing down, explaining and evaluating their predictions and listening to others' predictions can help students to begin evaluating their own learning and constructing new meanings. The discussion can also help teachers to provide appropriate feedback to students on their learning.

Teachers will also be able to link the Lulu game to the key competencies for Mathematical Practices such as making predictions; modelling (the use of multiple representations in enumerating the sample space); the use of tools to aid content exploration (the use of stones and the LuLu applet to gather experimental data); and a systematic approach to reasoning (listing elements of the sample space). As an extension to the LuLu challenge activity, students could be asked to investigate other cultural games such as Ashbii (McCoy et al., 2007). Computer simulations with the Lulu game can be used to demonstrate probability concepts, such as the shape of data, relative frequencies, and comparing simulated data with theoretical data. Students can then spend more time focused on making sense of data in various representations. A simulation tool created by Anna Fergusson is freely available at http://learning.statistics-is-awesome.org/modelling-tool-new/.

However, depending on the teachers' or the students' perspective, these computer simulations may favour one estimate of probability over another and issues related to sample size and the law of large numbers, or the difference between frequency and theoretical probability may or may not be discussed (Stohl, 2005). The emphasis on the experimental view may lead to a temptation to reduce teaching probability to the teaching of simulations-with little reflection on probability rules (Batanero et al.,
2016). We believe that designing effective learning activities that help develop probability knowledge and reasoning for a variety of students requires the integration of its different components of probability.
We strongly advocate the use of the Lulu game and other such culturally diverse games for probability explorations because such activities not only provide opportunities for students to learn probability concepts but also allow for simulations using both cultural artifacts and technological tools. In addition, these activities can help sustain student interest and motivation and help teachers highlight the significance of the role of culture and context in a mathematics classroom. The focus of this research was to help students learn the concepts of theoretical and experimental probabilities, sample space and possible outcomes using game context. We look forward to conducting future iterations of this research to test how consistent and useful these findings may be across diverse contexts. It is hoped that the findings reported in this paper will generate more interest in using game contexts in probability education. Teachers, curriculum developers and researchers need to continue to work together to find ways to help all students develop probability literacy.

## Acknowledgement

The project is funded by the Teaching and Research Initiative (TLRI) (2020-2022)

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